# FRACTAL ANALYSIS ON THE MINIATURE EROSION LANDFORM GENERATED BY ARTIFICIAL RAINFALL

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*Abstract* A miniature erosion landform was formed by applying artificial rainfall on a square mound made of a mixture of fine sand and kaolinite, and the surface topography was analyzed to evaluate the meaning of $H'$, which is a parameter expressing a certain complexity of self-affine surfaces, in the landform evolution. Erosion started as rapid valley erosion, and then slow degradation of interfluves and ridges followed. The average height shows a clear exponential decrease. After 192 hours of rainfall, a low and gentle topography appeared. The value of $H'$ decreases rapidly from 1 to 0.7~0.8 in the early stage and then gradually to 0.59 at the end of experiment, with a tendency to decrease towards 0.5 in a longer time period. This implies that the $H'$ value of landform tends to decrease towards 0.5 with a long period of erosion, and that some reiterated uplift would be necessary to keep the larger $H'$.

**Key words:** artificial rainfall, miniature erosion landform, fractal geometry, landform evolution

## 1. Introduction

Since Mandelbrot (1967) pointed out the statistical self-similarity of coast lines, landforms have been considered as a good example of fractal geometry found in nature. However, the standardized method to quantitatively express landform morphology with fractal geometry has not been established yet, although many attempts were conducted (Xu *et al.*, 1993). Matsushita and Ouchi (1989) pointed out that landforms have characteristics of self-affinity, and developed the method to analyze the self-affinity of various fractal curves including transect profiles of landform. A certain complexity of self-affine curves can be measured by the parameter $H'$, which is equivalent to the scaling parameter of fractional Brownian motion $H$ (Hurst parameter). This method was extended to analyze surfaces developing in three-dimensional space (Ouchi, 1990; Ouchi and Matsushita, 1992). The measured values of $H'$ on fractional Brownian surfaces well reproduced the values of $H$, with which these surfaces were generated (Ouchi, 1990). The $H'$ values

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measured on 1 : 25,000 scale topographical maps (*e.g.*, 0.72 for Yarigatake area, 0.61 for Tanzawa Mountains, and 0.48 for Yamizosan area) (Ouchi and Matsushita, 1992) indicate that high mountain areas have larger values of $H'$ and well dissected and lowered areas have values close to 0.5. The meaning of $H'$ value in the formation of landform, however, is not clear at all yet. This is because the actual development of erosion landform is virtually unknown. The development of erosion surface generated experimentally by artificial rainfall, on the other hand, can be observed and measured. It probably provides some ideas to explain the relationship between $H'$ and the landform evolution. In this study, the development of miniature erosion topography formed by artificial rainfall was measured and analyzed with $H'$ and other parameters, as a first step to evaluate the meaning of $H'$ in the landform evolution.

## 2. Equipment and Procedure

A square mound (about 100 × 100 × 20cm) was made of the mixture of fine sand ($D_{50}$ is about 0.12 mm) and kaolinite (15 : 1 by weight; coefficient of permeability $k=2.8\times 10^{-4}$cm/s). Artificial fine rainfall was applied on this mound from sprinkler tubes for agricultural use, four strips (1 m long each) of which were set around the mound about 1.2 m above the ground. Water was supplied from two water tanks set at the same height.

Generating uniform rainfall over the surface of mound was technically very difficult and precipitation varied widely in places, while the average precipitation was in the range about 25～50 mm/hr. The non-uniform distribution of rainfall determined the position of main valleys; however, this did not affect the change in overall characteristics of surface topography significantly. The sand mound was eroded by surface runoff, with occasional slope failure, in the early stage of experiment. The surface topography was measured by the point gage, which automatically reads and stores data ($x$, $y$ and $z$) in memory, 9 times after 1, 2, 4, 8, 16, 32, 64, 128, and 192 hours of rainfall. The rainfall was discontinued during the measurement (measurement of 77 sections took more than 30 hours). Erosion looked accelerated for a while when the rainfall was resumed, but this does not mean the change in the characteristics of topography. Measurements were made on the 77 measuring lines with 1 cm interval on the inner part of mound surface. Every break point of the cross section was measured by the point gage, and the data were later converted to 77 × 77 gridded data for the analysis.

## 3. Calculation of $H'$

In the case of self-affine transect profiles, the curve is divided into sections of equal distance by the yardstick method, and values for curve length $N$, standard deviations of $x$ and $y$ coordinates ($X$ and $Y$) of all measured points are obtained for each section. The respective average values of $X$, $Y$ and $N$ are regarded as the representative values for this yardstick length. $X$ and $Y$ are related to $N$ as:

$X \sim N^{\nu x}$; $Y \sim N^{\nu y}$.

$X$ and $Y$ are scaled with each other as:

$Y \sim X^{H'}$, and $H' = \nu_y / \nu_x$.

$H' = \nu_y$, because $\nu_x = 1$ for the curves without overlaps in $x$ direction like transect profiles of landform.

This method or "line scaling method" can be extended to self-affine surfaces ("area scaling method") (Ouchi and Matsushita, 1992). The surface is divided into squares of nearly equal surface area, applying a unit surface area as a scale. The elevation variance $Z^2$, basal area $A$ and surface area $S$ are obtained for each square unit. The respective average values are regarded as representative values for this scaling unit. $Z^2$ and $A$ are related to $S$ as:

$Z^2 \sim S^{\nu z}$; $A \sim S^{\nu A}$.

$Z^2$ and $A$ are scaled with each other as:

$Z^2 \sim A^{H'}$, and $H' = \nu_z / \nu_A$.

$H' = \nu_z$, because $\nu_A = 1$ for the surfaces without overlaps.

This area scaling method was exclusively used in this study.

## 4. Changes of the Miniature Erosion Topography and $H'$

Erosion started from the edges of the mound right after the rainfall began. After 1 hour of rainfall, some valleys up to about 10 cm deep developed on the surface (Fig. 1). The largest valley appeared on the southern edge of the mound, draining from north to south. The position of main valleys did not change throughout the experiment. The erosion of these valleys and their tributary valleys was intense in the first several hours, and then slowed down. The values of $Zi$, which is the average standard deviation of elevation in 10 × 10 cm square, increase rapidly in the first several hours and then decrease gradually (Table 1, Fig. 2). This reflects the rapid valley erosion in early stages and the later domination of slow degradation of interfluves and ridges. The remnants of flat original surface can be observed clearly in the block diagram showing the surface topography at 8 hours of rainfall, but they are obscure at 16 hours (see Fig. 1). At the end of experiment (192 hours of rainfall), valley widening and lowering of ridges made valleys unclear and the surface topography became gentle. The average height, *zmean*, shows a clear exponential decrease (Table 1, Fig. 3) with the overall average erosion depth of 109 mm; about 63,000 cm³ of material was eroded from the measured area. The value of $H'$ decreased rapidly from about 1 to 0.7~0.8 with somewhat unstable manner in the early stage, and it then decreased gradually to 0.59 at the end of experiment (Table 1, Fig. 4). The tendency of $H'$ change shown in Fig. 3 indicates that $H'$ would decrease gradually towards 0.5 if the experiment continued much longer.

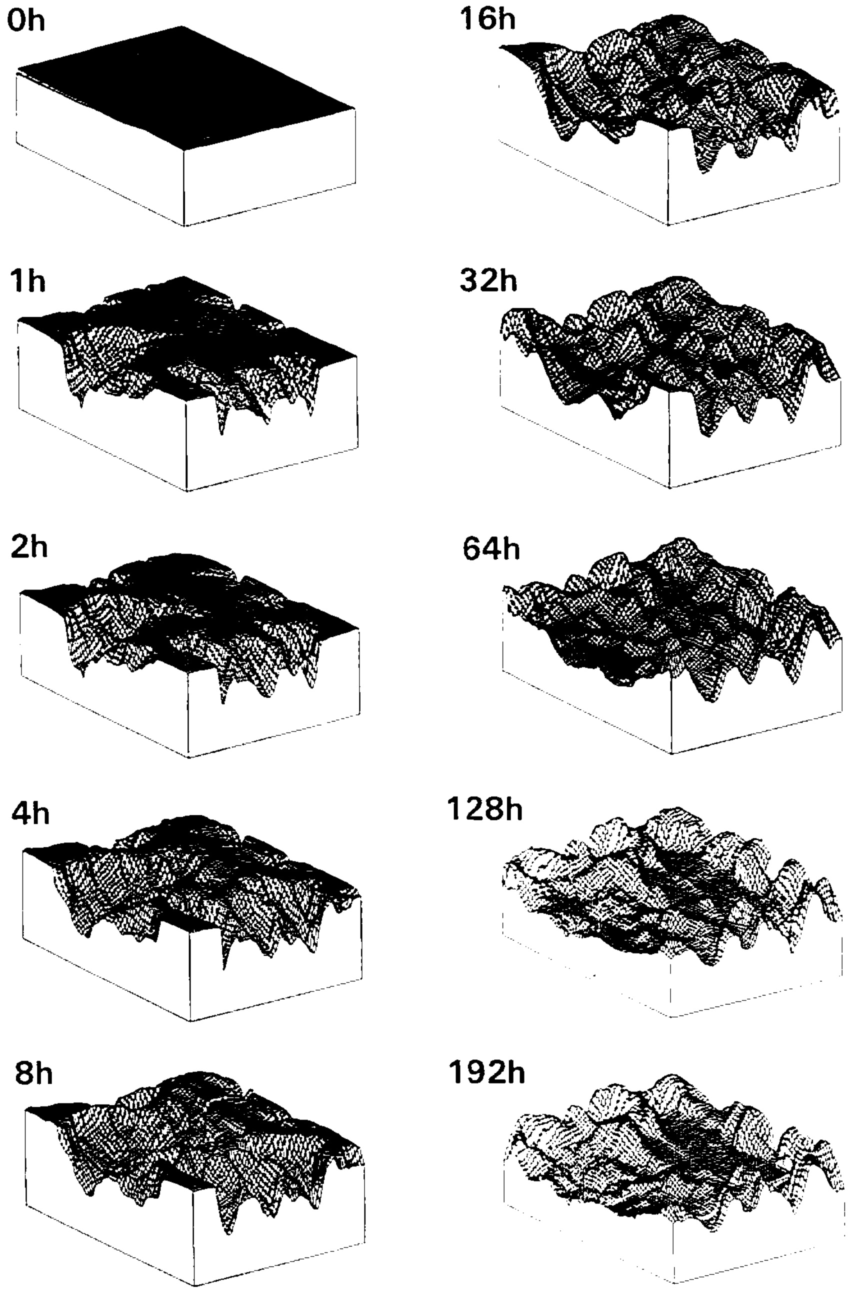


Fig. 1 Block diagrams showing the changes of surface topography of the mound with the time of artificial rainfall application
Side length of a block is 76 cm, and a short breaking line by the block indicates 10 cm of height. All the blocks are viewed from south-east direction. (Bottom lines of each block do not show the datum plane.)

Table 1 Values of average height (*zmean*), average standard deviation of elevation in a 10 × 10 cm square (*Zi*), and *H'*

| Time of rainfall (hours) | *zmean* (mm) | *Zi* (mm) | *H'* |
|---|---|---|---|
| 0 | 269 | 0.3 | 1.0 |
| 1 | 257 | 11.1 | 0.76 |
| 2 | 253 | 11.8 | 0.79 |
| 4 | 241 | 13.6 | 0.74 |
| 8 | 234 | 14.4 | 0.72 |
| 16 | 218 | 13.5 | 0.75 |
| 32 | 197 | 13.6 | 0.71 |
| 64 | 181 | 13.2 | 0.66 |
| 128 | 166 | 11.1 | 0.61 |
| 192 | 160 | 11.6 | 0.59 |

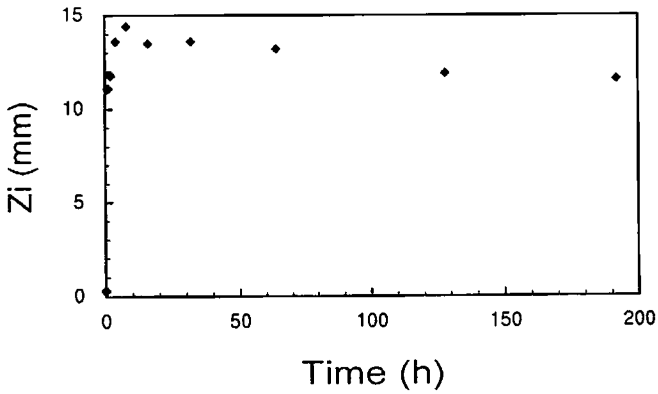


**Fig. 2** Changes of the average standard deviation of elevation in a 10 × 10 cm square, ***Zi***, with time

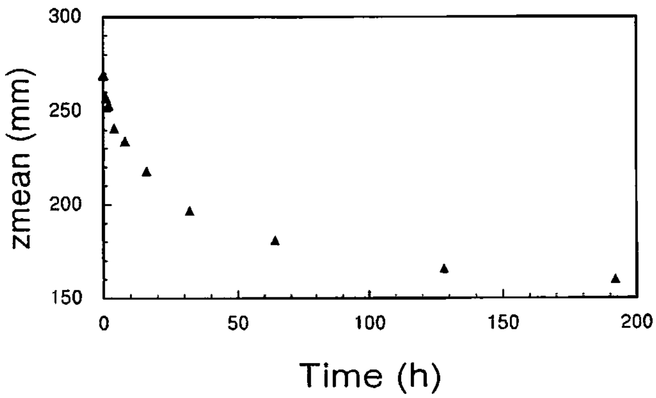


**Fig. 3** Changes of the overall average height, ***zmean***, with time

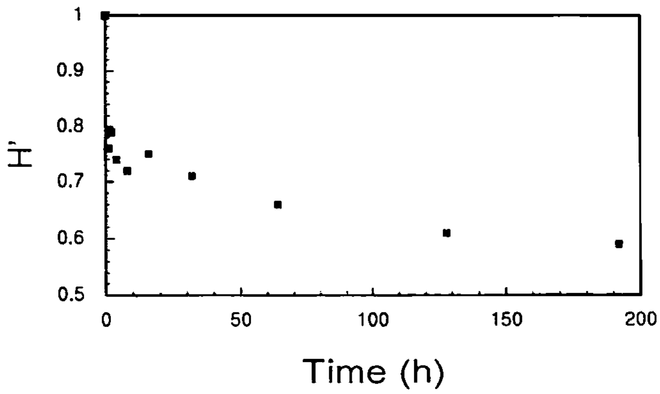


Fig. 4 Changes of $H'$ with time

## 5. Discussion

Czirók and Somfai (1993) made miniature landforms by spraying water on a smooth bank-like mound made of a mixture of silica sand and earthy soil with organic matter. They took a photo of the "profile" when a feature like real mountain ridge developed, and analyzed the digitized profile to get $\alpha$ value, which is equivalent to our $H'$. They pointed out that the value of $\alpha$ they got, $0.78 \pm 0.05$, agreed well with those obtained for the pictures taken from ridges in Dolomites, Italy ($0.8 \pm 0.1$). These values also agree with the values of exponent $H$ in the relationship that Hurst (1951) pointed out for the rescaled range $R/\sigma$ of some geophysical time series versus the period of record $n$, as $R/\sigma \sim n^H$, where $R$ is the cumulative sample range and $\sigma$ is the standard deviation of the data. Hurst (1951) showed that $H$ is about 0.7~0.8, instead of 0.5 that is predicted from statistic theories, and this has been called Hurst Phenomenon (Klemeš, 1974). The experiment conducted by Czirók and Somfai (1993), however, was relatively short in time, as it ended at the time when they saw the mountain like feature. If their experiment had continued for much longer time, their $\alpha$ value would have decreased. In the experiment of this study, the larger value of $H'$ such as 0.7~0.8 appeared in the early stage and decreased towards 0.5 in a long period of time with the progression of erosion. This seems to imply that the $H'$ value of landform has a tendency to decrease towards 0.5 with a long period of erosion, and that in order to keep the large $H'$ for a long period of time, some reiterated input of refreshing energy, namely uplift, would be necessary. The relatively large $H'$ value of 0.72 for Yarigatake area (Ouchi and Matsushita, 1992), which is characterized by high mountain ridges and deep valleys, seems to be concordant with this interpretation, assuming that the land surface has been eroded for a long enough period of time. The small $H'$ value for the well dissected Yamizosan area, 0.48, seems to reflect the long-term tectonic stability in this area. The property of material to be eroded, which is ignored in this discussion, certainly has some effects, and these will be studied in the next series of experiment. Klemeš (1974) concluded that the Hurst phenomenon is very complex to explain and cannot be attributed to one specific physical cause. The change of $H'$ values of miniature erosion surfaces in this experiment, however, seems to indicate that 0.5 may be the ultimate value of $H$ for landforms, and it provides one possible explanation about how the larger value of $H$ can be kept.

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