SIMULATION OF EVOLUTIONAL PROCESS OF LONGITUDINAL RIVER PROFILE USING EXPERIMENTAL FLUME AND COMPUTER

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Abstract In the present paper, the author intends to summarize his investigations on flume experiments and computer simulations of river profile development, focusing his attention on characteristics common to two simulators, flume and computer. The concentration of sediment to water discharge is set by a sand feeder and a water flow regulator. In the case of the computer simulation the sediment flux is given as the upstream boundary condition. As the flux is proportional to the slope, the boundary condition is defined by the slope. The downstream boundary condition is given by water level in the both cases, with some modification at performance of the simulation. A type of diffusion equation is introduced to describe transportation of sediments. This procedure is not required for the flume experiments, because flowing water carries sediments spontaneously. The fundamental equation is as follows:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left\{ke^{rx}\frac{\partial u}{\partial x}\right\}$$

where u is height of the river profile at point x and time t, k and r are constants. The r is an index expressing how sediment decreases downstream in size. The dynamic equilibrium state appears after sufficient time elapsed in the flume and also in the calculation result by computer. The steady solution of the equation above shows an exponential curve which is admitted widely to be the most adequate to longitudinal river profiles. In this equilibrium state the profile is determined by the independent external conditions derived from climate and/or sealevel. The change of external conditions makes fluvial landform develop to direction of a new equilibrium state determined by the new external conditions, and will result in formation of the alluvial plain or terracing.

Key words: computer simulation, flume experiment, river profile, mathematical model, dynamic equilibrium

1. Standpoint of View to Study

The main trend of studies on fluvial processes and landforms in Japan was over-viewed by Ikeda and Ouchi (1989) recently. One of the most interesting subjects is interpretation of river terrace development in the Quaternary. Effectiveness of chronological studies, field measurements of deposits, flume experiments and computer simulations on this theme have been proved already in their ways. In this paper I wish to look for a standpoint of view common to them. Speaking the conclusion at first, the basic concept is as follows.

The river is an open system, of which input and output are water discharge and sediment supplied from adjacent interfluvial slopes and released into the sea at the river mouth. The change of input or ratio of sediment to discharge produces change of the longitudinal river profile and results in formation of river terraces.

We find a thought of the open system in the early history of geomorphology. The basic thought was introduced by Gilbert (1877). Davis (1902) and his successors had developed it ideologically. The concept of 'grade' or balance of capacity and competence within the river system was established. For example, Mackin (1948) completed qualitative discussion on the equilibrium state of the river system. Strahler (1952) discussed dynamic open systems and the steady state citing von Bertalanfy's general system theory.

The new ways of thinking had been introduced to fluvial geomorphology by some researchers. For example, Hack (1960) introduced a quantitative treatment using geomorpho-metry, Leopold and his group (1964) considered it from a hydrological standpoint of view using fluvial observation data, and Schumm and his group (1987) showed effectiveness of experimental approaches.

The dynamic equilibrium theory of the open system that maintains itself in a steady state of the most efficient configuration by internal self-reguration has been very important concept in fluvial geomorphology. It plays a role to make us exclude mechanism, or hydraulics out of fluvial geomorphology. And it shows what are causes and what are effects for the long-term evolution of large-scale landforms such as river terraces.

Leopold and Maddock (1953) divided the variables of fluvial system into three categories: independent, semidependent and dependent. Once the dynamic equilibrium state has been attained in a fluvial system, the state of the system becomes to be a function of several independent variables such as sea level at river mouth, water discharge, and volume and size of sediments supplied, which are free from the state of system and determined by external conditions such as climate, vegetation, soil, topography, lithology, tectonic movement, world-wide sea level change *etc*. All hydraulic measures such as width, depth, velocity *etc*. of the water flow are semidependent or interdependent among them. Roughness of riverbed and channel pattern are also interrelated within the river system. We, geomorphologists who have interest in large-scale landforms rather than in micro-morphology on riverbed, look on the longitudinal river profile as the final dependent product of the all independent and semidependent variables.

Although we could not predict whether certain gravels on riverbed will move out or not by force of running water, we are able to get the functional relationship between gravel size and river gradient along a river course, for example. It is clear that we cannot derive the dependency of actual river profile on gravel size from knowledge on critical shear stress and critical velocity, or lift and drag forces, that is, hydraulics of transportation of each individual gravel on riverbed. In geomorphology, it should be explained at first which variables control the river profile and how they do before much effort will be spent on studies for hydraulic mechanisms of the river. It is sufficient, I believe, that the profile is defined functionally by the external independent variables. We can set up an audio amplifier using some devices of 'integrated circuit' without any knowledge on physical properties of semiconductors. Only input-output function as a black box is enough to use them. This understanding or viewpoint is validated by existence of the dynamic equilibrium in the river system.

2. Short Overview of Fluvial Landforms in the Japanese Islands

The Japanese islands are situated along a very active subduction zone of convergence of the global tectonic plates. High-relief mountains are located in the elongated islands. As the result rivers flow from mountains to the sea with short distance, forming alluvial fans at the base of mountains and small deltas in the bays. The rivers have abrupt longitudinal profiles and their bed sediments are gravels except in short reaches of the deltas.

River terraces are universal in the Japanese islands. Before the second World War, Japanese geomorphologists had believed that the terrace topography were formed by intermittent uplift of the islands. But now we know that crust movements persist with constant velocity not only on the average for a longer time than earthquake intervals, but also consistently for the whole Quaternary, and that the Quaternary climatic change has affected drastically fluvial conditions. Chronology of fluvial terraces has been well established in Japan using wide-spread tephra layers.

Only flash floods caused by intensive downpours of 'Baiu' in July and 'Typhoon' in September or October have capacity to form fluvial landforms. The rivers in the Japan Sea side of the islands are affected by snow-thaw floods too. The ratio of the maximum flood discharge to the average or minimum discharge in Japan is as high as in arid zones. It is believed widely that the Japanese main islands were not affected by deluges of 'Baiu' or 'Typhoon' in the Glacial period when the polar frontal zone should be displaced to south, although there are not sufficient and reliable evidence for the horizontal displacement of the climatic zones.

The forest line is located near the topmost peaks in Japan, where we find a level of cirque floors which coincides with the snowline altitude during the last Glacial period. Most rivers in Japan have at present no or negligibly small drainage area in the periglacial zone above the forest line. The distribution of vegetation and its related environments is controlled mainly by air temperature in Japan where precipitation is sufficiently enough for plant growth. Debris given to the rivers at present originates mainly from slope failures or debris avalanches due to intensive rainfalls in the coniferous and deciduous broad-leaved forest zones.

The Quaternary vertical displacement of climatic zones is estimated as one thousand several hundreds meters on the basis of snowline depression and fossil plant or palynological evidence. In conclusion change in the areal component of climatic zones within a drainage basin has affected discharge and sediment supply to the river and resulted in the terrace topography. Effect of climate and hydrology on the fluvial system

(Schumm, 1977) is still a main theme of fluvial geomorphology in Japan, although quantitative and theoretical studies remain in the future.

3. Fluvial Experiments in the Flume

The experiments made by Nogami *et al.* (1975a, b) and Sugitani (1985) aim to simulate geomorphological development of large-scale fluvial landforms such as alluvial plains or terraces. For this purpose we prepared a sufficiently wide flume (60 cm wide for discharge of 490 cc/sec at the maximum) so that 'alluvial plains' appeared along a channel in the flume. All experiments in which the channel width are equal to flume width such as the experiments by Gilbert (1914) or Guy *et al.* (1966) cannot simulate formation of the alluvial plain and its terracing. The experiments by Lewis (1944) are very similar to ours in the purpose of study. The flume used by him was four meters long and 50 cm wide for discharge of 66 or 40 cc/sec. He wished to simulate that substantial reduction in the supply of material from periglacial zone to rivers after the post-glacial climatic amelioration would lead to a rejuvenation and consequent terrace cutting in the upper reaches of the flood plains in Britain.

Nogami *et al.* (1975a) recognized existence of a threshold flume width which could affect the gradient of 'alluvial plain' in the flume. The experiments were carried out as follows. Uniform and noncohensive sand, 0.5-1.0 mm in diameter was used for the flume experiment. The gradient of a 'alluvial plains' and water surface, width and velocity of water flow were measured after sufficient time elapsed when a dynamic equilibrium state had been attained, that is, discharge of sand at flume end had continued to be equal averagedly to supply at flume head. This means there is no aggradation nor degradation in any reaches of the channel and 'alluvial plain'. But it was observed a small pulsation in amplitude of the ratio of sand supply to discharge, and this was recognized to exist in company with downstream movement of bed pattern in the channel and horizontal migration of the meandering channel itself in the flume.

Six RUNs were carried out for different 6 flume widths. The amount of sand and water supply were kept strictly constant during a RUN. The results are summarized in Table 1 and Fig. 1. The flume width determines the equilibrium gradient of 'alluvial plain' when the flume is so narrow that water width is limited by flume side wall. When the flume is sufficiently wide in comparison with water discharge, the steeper 'alluvial plain'

Table 1 Results of experiments (after Nogami *et al.*, 1975a)

RUN No.	discharge Q (cm^3/sec)	sand supply S (g/sec)	flume width (cm)	watcr width (cm)	velocity (cm/sec)	watcr depth (cm)	slope I ($\times 10^3$)
1	242	0.60	57.0	30.2	19.7	0.40	14.8
2	242	0.60	47.0	36.5	27.8	0.28	16.9
3	242	0.60	37.0	31.7	26.6	0.30	17.6
4	242	0.60	27.0	22.2	30.2	0.37	14.9
5	242	0.66	22.0	19.1	29.4	0.44	15.3
6	242	0.54	17.0	17.0	29.5	0.49	12.8

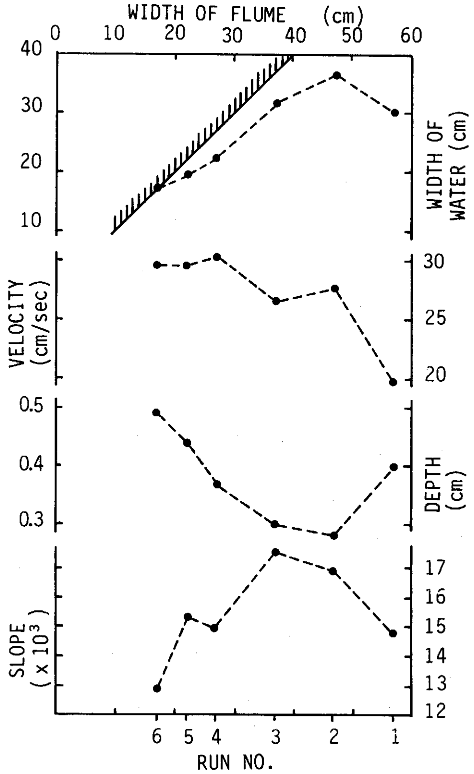

Fig. 1 Effect of flume width on velocity, width and depth of runnig water, and equilibrium gradient of 'alluvial plain' (after Nogami *et al.*, 1975a)

appears in the flume. In the latter case, the equilibrium gradient is determined mainly by discharge, independently of flume width. It is thus inferred that there is a threshold value of the flume width within which the gradient is affected by the flume width.

The purpose of our next experiments (Nogami *et al.*, 1975b) is to clarify the influence of water discharge and sand supply on the gradient of the 'alluvial plain' in an equilibrium state under fixed conditions of base level, flume declination and size of sand. They were carried out with the lower discharge than the maximum discharge at the threshold width, which was equal to the maximum width of our flume used.

Twenty-three RUNs of our experiments for various combinations of discharge and sand supply were carried out independently each other in order. In each RUN enough time was used for the appearance of equilibrium state. The results was shown in Table 2. Within the limits of our experiments, the equilibrium gradient is a linear function of sand supply (Fig. 2) and/or the ratio of sand supply to discharge (sediment concentration) when discharge is constant (Fig. 3). Figure 4 which is drawn from interpolated values using the regression equations in Fig. 3 shows sets of sand supply and discharge for a given gradient. Figure 3 says that changes of sediment concentration cause a channel to form 'alluvial plain' and terracing. As a matter of course, therefore, the basis of discussion on the cause of terrace topography should be quantitative.

Actual alluvial plains or terrace surfaces well-developed continuously for a long reach of rivers can be regarded as landforms built under the equilibrium state of that time. As concerns analogy between 'alluvial plain' in the flume and in the real world, we consider that the terrace surface (old alluvial plain) corresponds with 'alluvial plain' in equilibrium state of each RUN. Therefore, the simulation of continuous development of terrace topography was reconstructed only from logical interpretation of the results of RUNs under individual conditions. But Sugitani (1985) observed a transient state of the channel and 'alluvial plain' under a changing condition in the experimental flume. His experiments on the formation of 'fluvial terrace' under conditions of the tilting flume showed effectiveness of flume experiments for simulation of formation of terrace topography.

Table 2 Results of experiments (after Nogami *et al.*, 1975b)

RUN No.	discharge Q (㎤/sec.)	sand supply S (g/sec)	slope I ($\tan\theta \times 10^3$)
1	100	0.0716	27.2
2	100	0.257	33.3
3	100	0.633	36.1
4	100	0.747	39.9
5	100	1.09	49.9
6	242	0.257	21.0
7	242	0.635	25.5
8	242	0.758	25.9
9	242	1.12	31.7
10	242	1.95	37.0
11	242	2.25	37.7
12	242	3.10	42.5
13	340	0.262	21.0
14	340	1.14	27.0
15	340	2.13	33.8
16	340	3.73	38.2
17	490	0.257	21.5
18	490	0.730	23.2
19	490	1.15	24.4
20	490	2.20	29.0
21	490	3.03	31.2
22	490	4.52	33.1
23	490	5.67	37.7

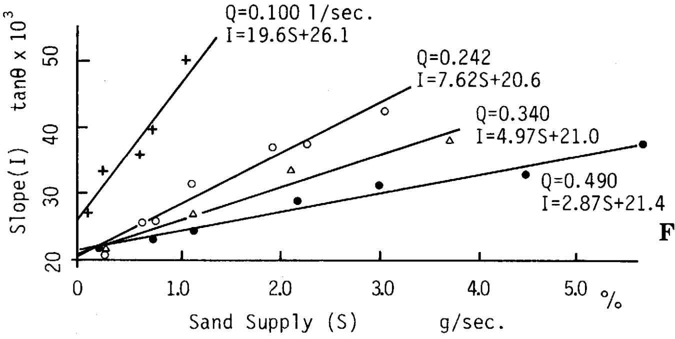

Fig. 2 Relation of sand supply to gradient at four discharge levels (after Nogami *et al.*, 1975b)

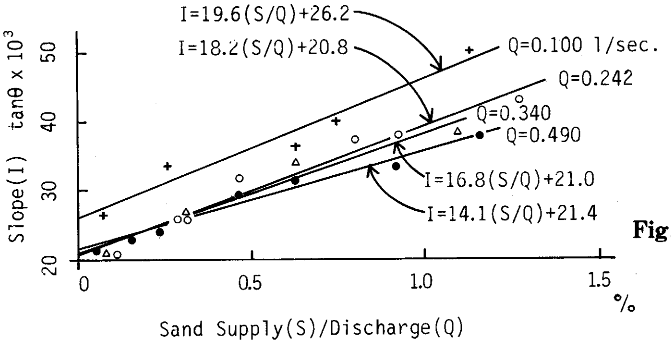

Fig. 3 Relation between ratio of sand supply to discharge and gradient (after Nogami *et al.*, 1975a)

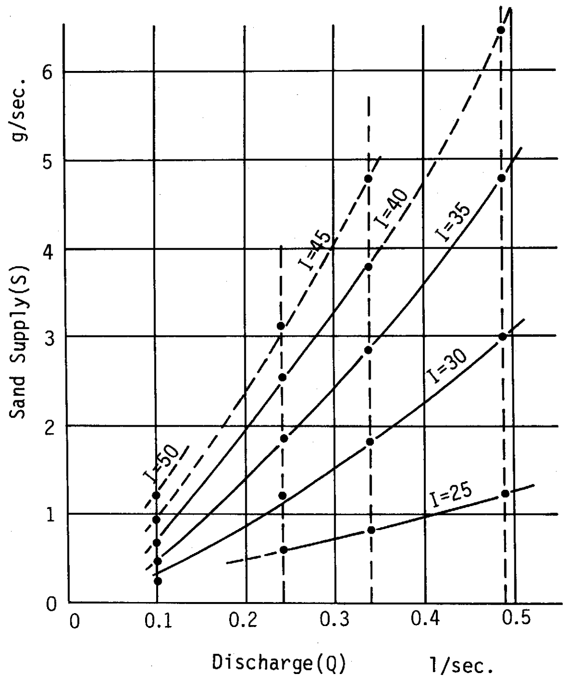

Fig. 4 Relation of sand supply to discharge at given gradient (after Nogami *et al.*, 1975a)

In conclusion, using the experimental flume we confirm the dynamic equilibrium (as defined ideologically by Mackin, 1948) under control of external conditions fixed for time. If there happens the change of external conditions (for example, sediment concentration) of a system in equilibrium, the state of the system such as river profile turns to change toward a new equilibrium state under the new conditions. Consequently terrace topography as a result of river profile changes will develop.

4. Computer Simulation

Instead of the experimental flume, Nogami (1981a) used a computer as simulator for the same purpose. Simulation by the computer has an advantage over by the flume in being more economical of time or labor, and in being able to set and change external and initial conditions more easily.

Fundamental model

Nogami (1981b) proposed a mathematical model of a type of the diffusion equation for the development of longitudinal river profile without confluence of tributaries. In this model, diffusion coefficient a is expressed as a function of distance x from the upstream boundary point of an objective river reach. Further discussions about the coefficient $a(x)$ are held over.

Sediment flux J per unit width is proposed to be proportional to the gradient ($\partial u/\partial x$, take note that the sign of this value is always negative) of the river profile:

$$J=-a(x)\frac{\partial u}{\partial x} \tag{1}$$

Differential of sediment flux with respect to x (spatial change of flux) should be equal to differential of height with respect to time (time change of height) if there is no volume change:

$$\frac{\partial u}{\partial t}=-\frac{\partial J}{\partial x} \tag{2}$$

The following equation is thus obtained from equation (1) and (2):

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left\{a(x)\frac{\partial u}{\partial x}\right\} \tag{3}$$

Using this equation of partial differential with respect to time and space, we can simulate the profile evolution of rivers.

Some discussions concerning equation (1) may be needed. In general, sediment flux J is expressed to be proportional to the mth power of the gradient I $(=\partial u/\partial x)$ of the profile as follows:

$$J\propto|I|^{m} \tag{4}$$

If m is not equal to 1 here, the equation (1 or 3) becomes nonlinear, and cannot be solved mathematically. Numerical method by a computer is an unique way for solution of the equation in this case. So I assumed for the sake of simplicity that m equals 1. As stated above, the equilibrium gradient in the experimental flume is in direct proportion to sediment flux within the limit of our experiments.

The most Japanese rivers are concave in longitudinal profiles even for the reach having no tributary confluence (Yatsu, 1955), where discharge can not increase downstream. Therefore, concavity must be caused by the effect of sediment. Sternberg, Shulits (1941), Yatsu (1955) and Hack (1957) thought that the gradient of river profiles was determined hydraulically in direct proportion to the size of gravel, which decreases downstream exponentially according to Sternberg's law. This is a very plain explanation but seems to be somewhat too mechanical. On the other hand, Leopold and Maddock

(1953) explained the concavity in terms of the downstream decrease of sediment flux in relation to discharge. In the case of the Japanese rivers mentioned above, the decrease of flux must be introduced by downstream fining of gravel because there is no confluence for the objective reach.

The present author (Nogami, 1981b) adopts a third idea that gravel size controls the profile through the diffusion coefficient. This idea enables to simulate dynamic (time-dependent) development of river profiles as will be stated later.

The concave profile may be attributed to: 1) a decrease of effective sediment flux, or 2) an increase of diffusion coefficient, with downstream fining of gravel. In the Case 1), the proportional coefficient of the equation (2) is not 1 but less than 1, because there is a loss of sediment by attrition. This coefficient may be expressed to be a function of distance x. In the Case 2), diffusion coefficient $a(x)$ of the equation (1) is already defined as a function of the x. Even if we have an alternative choice, the two cases meet and result in the same form of equation (3). Selection of consideration is offered freely, but it depends on which is more logical for thinking, more theoretical or more useful for mathematical treatment.

Generally speaking, the sediment flux measures more difficult than gravel size in an actual river. Other than this, to know paleohydrological conditions of rivers we can get information easily on gravel size of terrace deposits, but never on sediment flux at the time. Therefore, the author (Nogami, 1981b) prefers a type of the diffusion equation (3) as a fundamental model, because the measurable gravel size can be expressed explicitly in the diffusion coefficient.

Please see the original texts (Nogami, 1981b, c) for further discussion on this model, in which it was explained how the diffusion coefficient as a function of the travel distance was introduced from a dynamic Markov process.

Diffusion coefficient

The downstream fining of the gravel size on riverbed is made by abrasion and partition of gravels being transported selectively after the size. This process can be simulated in a rotating barrel. It depends on rock property and gravel size itself which is dominant for the gravel fining, abrasion or partition. Gravel size is a function of the assumed distance traveled proportional to the number of rotations (Moriwaki *et al.*, 1984).

It is natural to admit that the diffusion coefficient (in the other expression, easiness of transportation) increases downstream in inverse proportion to gravel size. The following equation that defines the diffusion coefficient is rationally introduced from Sternberg's law.

$$a(x)=ke^{rx} \tag{5}$$

where k and r are constants. The value r is influenced by the rock property and transportational processes of gravel on riverbed. Adequateness of this definition will be discussed later.

Boundary conditions

Obviously the river mouth is the lower boundary where the river delivers fluvial

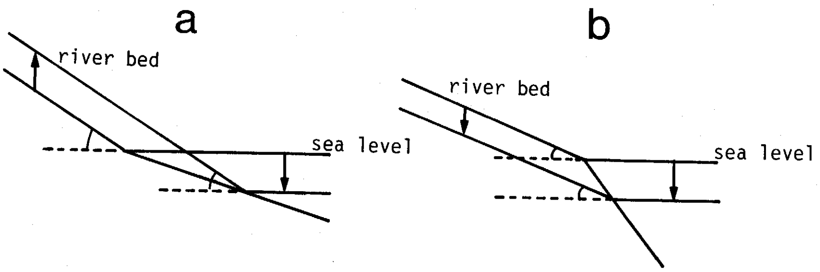

Fig. 5 Fluvial deposition (5-a) and deepening (5-b) caused by sealevel lowering (after Nogami, 1981a)

sediment to the sea. The gradient of a river at the mouth should not be zero because sediment flux is not zero there by definition of the equation (2). We can define a boundary condition at the mouth in the two manners, by potential and/or by flux. Potential at the river mouth is equal to the sea level which changes with time and is represented by a function of time. For performance of simulation we can utilize the values read from the curve of world-wide sea level change. This is a very explicit way to understand if the river mouth is forced to stand at the same point during sea level change, such as if a river is open directly to abrupt continental slope. But in the mountainous Japanese islands, a considerable number of rivers have abrupt profiles which continue to the more gentle slope of continental shelves under the sea level. In this case lowering down of the sea level produces deposition for the lowermost river reach to the mouth which is shifting away (Fig. 5). Thus setting the sea level at the boundary fixed in position is unrealistic. On the other hand, as the sediment flux is determined by upstream conditions, the gradient at the river mouth must be maintained independently from the sea level change.

In the simulation made by Nogami (1981a) the boundary condition was defined by the gradient, that is, by the sediment flux at the river mouth, of which position was passively floating following altitudinal relationship between the sea level and the profile of river/sea-bed. The latter altitude changes automatically within the river system, and the former changes independently from the state of the system. In the other expression, the sea level is defined independently, but position of river mouth is determined by the flux from upstream. The values of sediment flux used for simulation were calculated from the slopes at the river mouths estimated in the profiles of terrace surfaces of different ages (Nogami, 1981a). Difference between sediment supply at the upstream boundary and sediment discharge at the downstream boundary is a loss of effective sediment flux made by abrasion for this river reach.

The upstream boundary condition should not be given by potential but by sediment flux, that is, by the gradient. The boundary can be set at any point of river reaches if there is no confluence. The sediment flux supplied at the boundary is independent from the state of river and determined mainly by climate and its related conditions such as vegetation and soil. Geology and geomorphology of the drainage basin decide the flux, but they does not change for the time span of terrace formation.

Initial conditions

To perform a simulation for the dynamic (time-dependent) development of river profile, we need an initial profile from which the simulation will start. An exponential profile is fitted to the well-developed terrace surface, using the least square method. In the simulation made by Nogami (1981a), it was used the Musasino Terrace of the Tama River in Kanto Plain, whose formation is dated as belonging to the early Wurm when the sea level stood still high. Then the initial altitude at each point needed was calculated from the equation of the profile. This procedure corresponds quite with preparation of an initial surface of sand in the flume. If the equilibrium profile is once attained, the initial condition has no influence on a following development of the profile.

Equilibrium state

When time goes to infinity under constant boundary conditions, the change of height with time becomes to cease. In this equilibrium (steady) state, the sediment flux is constant everywhere for all reaches of river. In mathematical procedure, we substitute 0 for the left side of the equation (3), or a constant value (equilibrium flux) for the left side of the equation (1). The steady solution of the equation (3) with (5) is given in a form of the exponential curve as follows:

$$u = C_0 e^{-rx} + C_1 \tag{6}$$

where C_0 and C_1 are integral constants. If we differentiate both sides of the equation (6), we get an equation of the equilibrium slope which is exponential too.

The exponential curve has been one of the most adequate mathematical expression for river profiles (Shulits, 1941; Yatsu, 1955). Woodford (1951) and Morisawa (1968, 1985) summarized several examples of empirical equations. Logarithmic curves are popular besides exponential. Jones (1924) and Hack (1957) formulated a logarithmic equation for river profiles. But I cannot agree on that the logarithmic equation is fundamental because it describes only a look of the profile and lacks theoretical basis.

Some steady solutions are derived mathematically from various types of equations with certain conditions (Table 3), some of which are unrealistic and have no theoretical basis. They are rectilinear, catenary, exponential, logarithmic, parabolic and power functions. Among them we find the two equations which give an exponential curve as a steady solution. One of them is adopted as the fundamental model in this paper. The other is the same equation that Hirano (1966, 1989) introduced for interfluvial slopes. The change of height with time is expressed to be the sum of first and second partial derivatives of height with respect to distance as follows:

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial u}{\partial x} \tag{7}$$

where a and b are constants. Here we must take note that if the river mouth is set at the left/right hand in the longitudinal profile, b must be a negative/positive value as the equation might be applicable to the concave river profile. This restriction is caused by that the slope ($\partial u/\partial x$) is vector but the second partial derivative of u with respect to x

Table 3 Some mathematical models and their steady solutions for the longitudinal river profiles (after Nogami, 1981b)

a, b, $r=b/a$, m, n, and k are constants, C_1 and C_2 are integral constants.

fundamental esuation	condition	steady soution	remarks
1) $u_t=a\,u_{xx}+b\,u_x$	$b=0$	$u=C_1x+C_2$	rectilinear
	$b\neq 0$	$u=C_1\exp(-rx)+C_2$	exponential
2) $u_t=ax^m\frac{\partial}{\partial x}(u_x)^n+bx^{m-1}(u_x)^n$	$b=0$	$u=C_1x+C_2$	
	$n=r\neq 0$	$u=C_1\ln x+C_2$	logarithmic
	$n\neq r$	$u=C_1x^{(1-r/n)}+C_2$	power function
3) $u_t=a^2u_{xx}-b^2u$	$ab\neq 0$	$u=C_1\exp(rx)+C_2\exp(-rx)$	catenary
4) $u_t=\frac{\partial}{\partial x}\left\{a(u)\,u_x\right\}$		$C_1x+C_2=\int^u a(\tau)d\tau$	
5) $u_t=\frac{\partial}{\partial x}\left\{a(x)\,u_x\right\}$		$u=C_1\int^x\frac{d\tau}{a(\tau)}+C_2$	
	$a(x)=a$	$u=C_1x+C_2$	rectilinear
	$a(x)=ax+b$	$u=C_1\ln\|x+r\|+C_2$	logarithmic
	$a(x)=(ax+b)^2$	$u=C_1/(x+r)+C_2$	parabolic
	$a(x)=ax^m$	$u=C_1x^{-m+1}+C_2$	power function
	$a(x)=k\exp(rx)$	$u=C_1\exp(-rx)+C_2$	exponential

is scalar. Expansion of this one-dimensional model to two-dimensional one is prevented by this restriction. It can be said that equation (7) has a week point in its physical basis.

The equation (7) describes the evolutional development of slope form (change of height with time), but can hardly express sediment flux, because geomorphological analogue of the second term of right side is not obvious concerning flux, and the supposed flux caused by the second term conflicts with diffusion flux by the first term (Nogami, 1981c). As the spatial change of sediment flux (aggradation and degradation at a point) is essentially important to cause landform to develop, the fundamental equation should be efficient to express both landform and material transportation.

Result of simulation of terrace development of the Tama River

The equation (3) with (5) was applied to simulate the terrace development of the Tama River, which is originated from Kanto Mountains, enters the depositional Kanto Plain forming alluvial fan (whose main parts are terraced at present) and ends at Tokyo Bay. Its *ca.* 1,000 km^2 drainage basin with a high relief is roughly estimated to discharge *ca.* 1.5 × 10^{-4} km^3/yr of sediment. The well developed terraces of the Tama River studied by the Kaizuka (1958) have been chronological types of the Japanese late Pleistocene and Holocene.

The simulation (Nogami, 1981a) started from the best developed Musasino Terrace surface (about 60 × 10^3 yrBP) whose profile was given as the initial condition. The height of sea level at the stage of each terrace surface was roughly estimated from various data at the suggestion of Prof. H. Machida of our Department. The constant r of the equation (5) was calculated using the least square method from each longitudinal profile of the terraces expressed as the equation (6). As far as the sediment flux we have very few information on the upstream boundary condition. The flux was very roughly estimated from the gradient of the profile and gravel size at the boundary (Table 4). The total sediment flux for the Holocene was checked by the volume of Holocene fill deposits in the valley topography which had formed till the lowest sea level at the end of the Pleistocene.

The results of simulation well represent the actual development (change with time) of river profile, which is stated by the early Wurm fill-strathing in the upper-most reach of the River, the latest Pleistocene over-flowing on the older fan surfaces in the upper-middle reach, the forming of valley topography in the lower reach due to the maximum sea level lowering and of drowned valley topography due to the postglacial sea level rising, and then river mouth advancing toward sea caused by fluvial accretion during the last seven thousand years.

In the Fig. 6 an abrupt rise of riverbed height implies valley filling with deposition, and an abrupt fall does forming valley topography with cutting down. While the height is changing slowly, the strath surface which has veneer deposits will be formed. Fill-strath terraces (such as multiple set of the low-cliff Tachikawa Terraces of the Tama River, $40 \sim 20 \times 10^3$ yrBP) are widely distributed in Japan. The change of height of riverbed with time is different according to position of the river reach (distance from upstream boundary). For example the riverbed height at the upstream boundary reached to the maximum in 40×10^3 yrBP, but at the point of 20 km downstream did in 15×10^3 yrBP. This means that there was a crossing of two terrace surfaces in the longitudinal profile. The other crossing common for almost rivers in Japan happens between the actual alluvial plain and the terraced fan surface formed in the sealevel lowering maximum.

We can draw a multiple set of profiles of different time using calculated values as results of the simulation (Fig. 7). But can we draw a profile of certain time in real world? The answer is clearly 'No' because we have no knowledge about whether downcutting started at the same time along the whole reach of river. We reconstruct the old river profile using terrace surfaces distributed in places on the assumption that well developed (paired) terraces will continue smoothly upstream and downstream. The results of simulation suggests that terracing is not always caused by rapid upstream shifting of a profile convexity, but in many cases it starts at the upstream boundary or in the middle river reaches. Using in detail tephrochronological method, Yanagida (1981) reported a good sample that the downcutting was initiated in the middle course and followed by slow shifting to both upstream and downstream reaches along the Saru river of middle

Table 4 Some values calculated and estimated for the Tama River and its terrace surfaces (after Nogami, 1981a)

	present	buried valley bottom	T-terrace	M-terrace
sea level (m)	0	−135.43	−33.60	−13.11
river mouth (km)	49.4	88.0	64.1	64.4
gradient at river mouth ($\times 10^{-3}$)	−1.0321	(−1.0321) −1.70	−1.0321	−1.0321
shape index $r = b/a$ (km^{-1})	0.04016	0.01413	0.03329	0.02661
height at the upper boundary (m)	161.2	161.6	196.9	163.9
gradient at the upper boundary ($\times 10^{-3}$)	−7.506	−5.898	−8.705	−5.742
equilibrium sediment flux ($\times 10^{-4} km^3/yr$)	$\simeq$1.5	$\simeq$5.8	$\simeq$3.0	$\simeq$1.5

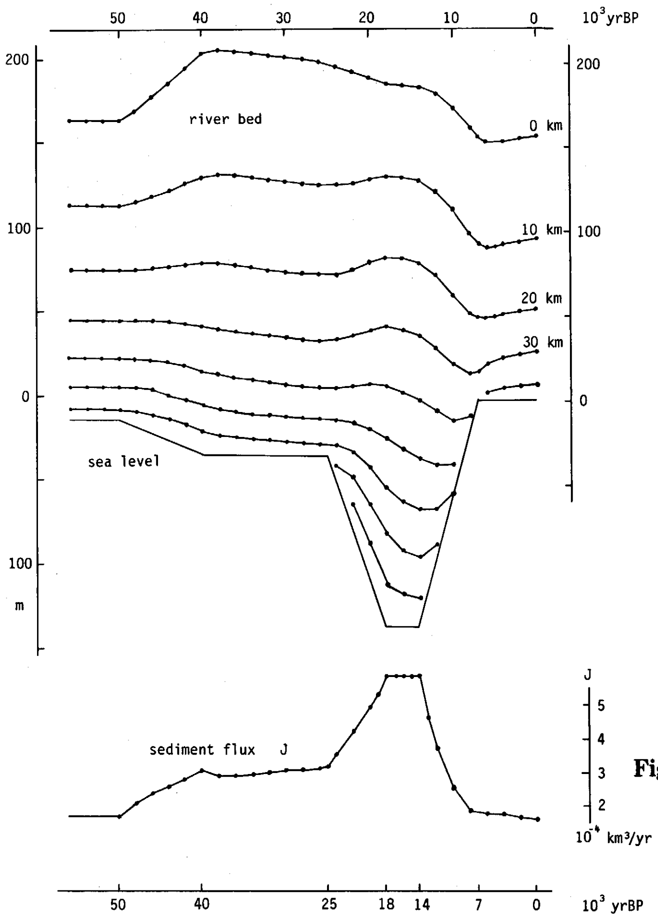

Fig. 6 Result of simulation: change of river bed height with time at different points of distance from upstream boundary (after Nogami, 1981a)

Hokkaido. This tells us that in detail dating of terrace surface by tephra layers is a key point of the study on how terrace topography develops in the real world.

5. Concavity of Longitudinal River Profile

Concavity of the profile differs among rivers in general. It depends mainly on mechanical or physical properties of gravel, and/or characteristics of discharge, and/or types of transportation such as debris flow or traction of one by one. In the equation (5) gravel size is expressed to be inverse proportion to the diffusion coefficient. The concavity r indicates how rapidly sediment decreases downstream in size. It is interesting to be able to detect climatic influence on the concavity from large local varieties mainly due to geology and discharge. After my observations in the Andes, the concavity in the humid tropical zone is stronger than in the semi-arid or periglacial zones.

It is widely admitted in Japan that the terrace profile of the latest Pleistocene when the sealevel lowering reached at the maximum shows weaker concavity than the others,

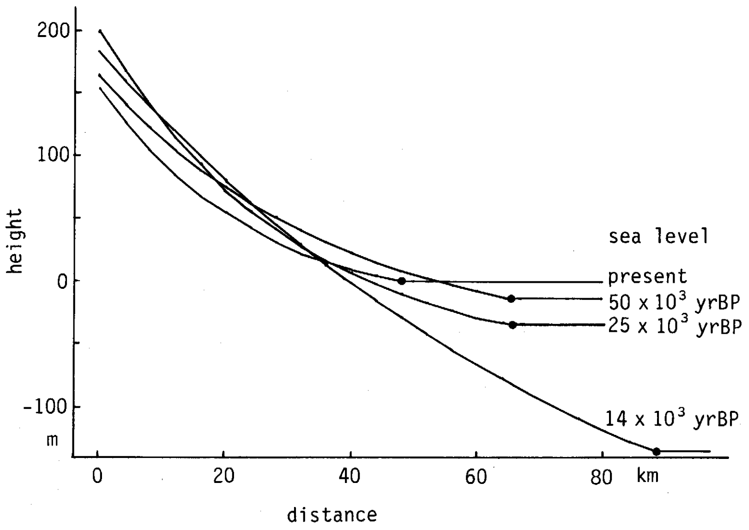

Fig. 7 Result of simulation: longitudinal profiles at different ages (after Nogami, 1981a)

especially the present river profile. The concavity, therefore, changes with time for the same river. As the inherent properties of a drainage basin such as geology or topography does not change for time, the change of concavity should be influenced by climate. We have very few knowledge on how climate influences concavity of the river profile. Further investigations on this theme are required in future.

6. Some Concluding Discussions

The simulations on terrace formation in the experimental flume (Nogami *et al.*, 1975b; Begin *et al.*, 1981; Gardner, 1983; Sugitani, 1987) are suggestive for us to get an idea or concept on developmental process of fluvial landforms. Also the rapid motion cinema of terrace formation in the flume is very educative especially for beginners of geomorphology. If a practical use is intended, limitation of this approach exists clearly in quantitative similarity between phenomena in the flume and in the real world.

The computer simulation is a better labor-saving approach than the flume experiment because conditions can be easily set up and changed. Besides the time needed for a RUN is distinctly shorter. A fundamental equation in the case of computer simulation is resembles action of flowing water in the flume. It describes how sediment is transported by flowing water. To solve the equation mathematically or numerically we are required to set boundary conditions. This leads us to understand a geomorphological process as an open system. Through the performance of computer simulation we are forced to recognize what is essential for geomorphological study and what is lacking in our knowledge on geomorphological processes. This may be the most important advantage of the computer simulation.

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