EFFECT OF CHANGING OBSERVATION TIME ON MEAN TEMPRATURES

Takuzo HOHGETSU

INTRODUCTION

Changes of location of a meteorological station, meteorological instruments and calculating formulae of observed data cause inhomogeneity of climatic data. In researches as of climatic change, which deal with a long-term climatic data series, degrees of changes in climatic data may be vital. It is, however, unknown whether the degrees of changes may be true or seeming, if the inhomogeneity is ignored. Degrees of changes in climatic data caused by changes of location of a station and the others should be evaluated in such researches.

In many researches degrees of the changes have been evaluated simply by comparing means, regression equations or others of two climatic data series (for example, Statistical Section, JMA 1961). In comparison with means such as monthly mean, comparatively short-term means as five-, or ten-day means have larger variances in mean. Therefore, it may be more difficult to evaluate degrees of the changes by this kind of methods. In researches to ensure homogeneity of data, stochastic methods such as run tests (for example, Thom 1966) have been applied on the basis of probability of occurrence. To evaluate more precisely degrees of the changes of data with large variance, it may be also valid to employ this kind of methods. In this research the author tried to evaluate in terms of probability degrees of the changes of comparatively short-term mean temperatures which were caused by changes of calculating formulae.

Five-, ten-day and monthly mean temperatures, which are dealt with in this research, are calculated by averaging daily mean temperatures. Therefore, changes of formulae for calculating daily mean temperature may be essential causes which bring the inhomogeneity investigated in this research. Up to now since the establishment of Japan Meteorological Agency (JMA) in 1875, there have been four principal changes of formulae for calculating daily mean temperature. Those are (a) averaging twenty-four observed temperatures at 01, 02, 03, . . . 23, 24 Japan standard time (JST), (b) averaging eight temperatures at 03, 06, 09, 12, 15, 18, 21, 24 JST, (c) averaging six temperatures at 02, 06, 10, 14, 18, 22 JST, and (d) averaging three temperatures at 06, 14, 22 JST. Since 1953 JMA has adopted formula (b) for calculating daily mean temperature.

In this research daily mean temperatures were calculated by the formulae (a), (b), (c) and (d) (${}_1T_{24}$, ${}_1T_8$, ${}_1T_6$, ${}_1T_3$). Then on the basis of these daily mean temperatures, deviations of ${}_5T_{24}$, ${}_5T_6$, ${}_5T_3$ from ${}_5T_8$, of ${}_{10}T_{24}$, ${}_{10}T_6$, ${}_{10}T_3$ from ${}_{10}T_8$, and of ${}_{31}T_{24}$, ${}_{31}T_6$, ${}_{31}T_3$ from ${}_{31}T_8$ were tried to be respectively evaluated in terms of probability.

Abbreviations: JMA: Japan Meteorological Agency, JST: Japan standard time, iTj: i day mean temperature on the basis of daily mean temperatures which are derived from data of j time observations a day.

DATA

Temperature at Maebashi meteorological station (36.24N, 139.04E) has been observed every hour, and registered on the original record of JMA. In this research the data of every hourly temperature at the station were taken for twenty-five Januaries since 1951 to 1975. Daily mean temperatures were calculated by the formulae (a), (b), (c) and (d). Maebashi mateorological station did not move during this period.

METHODS

Probability of Occurrence of k Days with Plus (or Minus) Deviation in n Consecutive Days

It was assumed that a day with plus (or minus) deviation from ${}_1T_8$ in daily mean temperature occurres according to a Markov chain.

A sequence of trials with possible outcomes $E_1, E_2, \ldots$ is called a Markov chain if the possibility of sample sequences is defined by $P[(E_0, E_1, \ldots, E_n)] = a_0 P_{01} P_{12} P_{23} \ldots P_{n\text{-}1n}$ in terms of a probability distribution $[a_k]$ for E_k at the initial (or zero-th) trial and fixed conditional probability P_{jk} of E_k given that E_j has occurred at the preceding trial (Feller 1957). A Markov chain can be determined by transition probability, P_{jk}, and $[a_k]$. Maximum likelihood estimates for P_{jk}, subject to the restriction to

$P_{jk} \geqq 0,\ \sum_{k=1}^{m} P_{jk} = 1\ (j = 1, 2, \ldots, m)$, are

$$\hat{P}_{jk} = \sum_{t=1}^{T} N_{jk}(t) \Big/ \sum_{t=0}^{T-1} N_j(t) \qquad \cdots\cdots (1)$$

here $N_{jk}(t)$ is the number of individuals in state j at t-1 and in state k at t (Anderson and Goodman 1957).

In this research two elemental states were adopted, that is, days with plus and minus deviations from ${}_1T_8$. The number of days with no deviation from ${}_1T_8$ were five for ${}_1T_{24}$ and ${}_1T_6$ and four for ${}_1T_3$. Those were so small in number to the number of total days (775 days) that events of days with no deviation were eliminated in calculating probability.

Probability of occurrence of k days with plus (or minus) deviation from ${}_1T_8$ in n consecutive days $(n \geqq k)$ can be calculated by two-state Markov chain model (Gabriel and Neumann 1962). In this research this probability was calculated by the computing method shown by Katz (1974).

Probability of Occurrence of Total Deviating Amounts in n Consecutive Days

By two-state Markov chain model probability of occurrence of total deviating amounts in n consecutive days can not be calculated. In this research it was assumed that a relative frequency of occurrence of a plus (or minus) deviating amount in the actual sample of days with plus (or minus) deviations was probability of occurrence of the deviating amount of a day with plus (or minus) deviation. On this assumption probability of occurrence of total deviating amount (d) in n consecutive days containing k days with minus deviation were calculated. Let this probability be D(n, k, d). D(n, k, d) is subject to the following restriction

$$\sum_{k=0}^{n} \sum_{d=b}^{a} D(n, k, d) = 1$$

here maximum plus and minus deviating amounts are α and β, respectively, and a and b are α (n-k) and βk, respectively. In this research D (n, k, d) was calculated by the following way,

$$D(n, k, d) = D_m(k, x) \cdot D_p(n\text{-}k, y)$$

here $d = x + y$

D (n, k, d) : probability of a total deviating amount, d, in the case of k days with minus deviation in n consecutive days.

$D_m(k, x)$: probability of a total deviating amount, x, in the case of k days with minus deviation.

$D_p(n\text{-}k, y)$: probability of a total deviating amount, y, in the case of n-k days with plus deviation.

$D_m(k, x)$ and $D_p(n\text{-}k, y)$ were calculated on the above mentioned assumption.

Probability of occurrence of total deviating amounts in n consecutive days can be calculated by the following equation

$$Td(n, d) = \sum_{k=0}^{n} D(n, k, d) \cdot M(n, k) \quad \text{.........................} \quad (2)$$

here Td(n, d) is probability of occurrence that total deviating amount is d in n consecutive days, and M (n, k) is probability of occurrence of k days with minus deviation in n consecutive days. As mentioned before, the latter probability was calculated by two-state Markov chain model.

RESULTS

Transition Probability and $[a_k]$

Maximum likelihood estimates for transition probability, $\hat{P}_{11}$, $\hat{P}_{12}$, $\hat{P}_{21}$ and $\hat{P}_{22}$ were evaluated by using the equation (1) (Table 1). Here subscripts 1 and 2 refer to plus and minus deviations, respectively. $\hat{P}_{12}$, for example, denotes probability of occurrence of a day with minus deviation, given plus deviation on the previous day. A probability distribution

Table 1 Maximum likelihood estimates for transition probability and $[a_k]$.

Suffixes 1 and 2 denote plus and minus deviations, respectively.
(A): in the case of 24 time observations a day, (B): 6 time observations,
(C): 3 time observations

	$\hat{P}_{11}$	$\hat{P}_{12}$	$\hat{P}_{21}$	$\hat{P}_{22}$	a_1	a_2
(A)	0.3900	0.6100	0.4386	0.5614	0.4183	0.5817
(B)	0.4637	0.5363	0.4895	0.5105	0.4772	0.5228
(C)	0.4929	0.5071	0.4524	0.5476	0.4715	0.5285

$[a_k]$ for event E_k. at the initial (or zero-th) trial was determined by the following equations (Gabriel and Neumann 1962)

$$a_1 = \frac{\hat{P}_{21}}{\hat{P}_{12} + \hat{P}_{21}}, \qquad a_2 = \frac{\hat{P}_{12}}{\hat{P}_{12} + \hat{P}_{21}}$$

here a_1 and a_2 are absolute probability of days being of plus and minus deviations, respectively.

Fit of the Model

The fit of the Markov chain model was examined by testing whether the proportions of a day with minus or plus deviation, given the previous day's deviation, were independent of the deviation two days earlier. Chi-square tests (Anderson and Goodman 1957) were not significant at the 5 per cent level (Table 2), so that the model might be said to hold.

Table 2 Occurrence of days with plus and minus deviations on two preceding days.

(A): in the case of 24 time observations a day, (B): 6 time observations,
(C): 3 time observations

	Preceding days		Actual day (days)		
	Second	First	Plus	Minus	Total
(A)	plus	plus	34	78	112
	plus	minus	69	108	177
	minus	plus	75	106	181
	minus	minus	107	133	240
(B)	plus	plus	65	92	157
	plus	minus	85	99	184
	minus	plus	88	94	182
	minus	minus	98	92	190
(C)	plus	plus	79	88	167
	plus	minus	88	86	174
	minus	plus	86	86	172
	minus	minus	81	119	200

Tests of independence of second preceding day

$x^2 =$ 5.730 with 2 d.f., P = 0.05 – 0.10 for (A)
2.929 with 2 d.f., P = 0.10 – 0.25 for (B)
4.106 with 2 d.f., P = 0.10 – 0.25 for (C)

Probability of Occurrence of k Days with Minus Deviation in 5, 10 and 31 Consecutive Days

Probability of k days with minus deviation in 5, 10 and 31 consecutive days were calculated, respectively (Figs. 1, 2 and 3).

Probability of k days with minus deviation in 5 consecutive days were shown in Fig. 1. It became clear that in all the cases of 24, 6 and 3 time observations, probability of three days with minus deviation was the largest and the second largest was that of two days with minus deviation, and that a probability distribution of 6 time observations was very similar to that of 3 time observations.

Fig. 2 showed probability of k days with minus deviation in 10 consecutive days. In the case of 24 time observations, probability of six days with minus deviation was the largest. On the other hand, in the cases of 6 and 3 time observations, probability of five days with minus deviation was the largest. Both probability distributions were similar each other and those were similar in shape to that of 24 time observations displaced leftwards.

Fig. 3 showed probability of k days with minus deviation in 31 consecutive days. There were similar features as in Fig. 2. In the case of 24 time observations, probability of eighteen days with minus deviation was the largest and a probability distribution was symmetrical in shape. On the other hand, in the other two cases probability of sixteen days with minus deviation was the largest. Both probability distributions were similar each other, and those were similar in shape to that of 24 time observations displaced leftwards by two days.

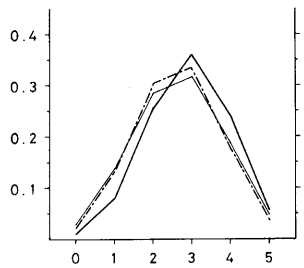

Fig. 1 Probability of days with minus deviation in 5 consecutive days.

Ordinate: probability, abscissa: the number of days with minus deviation. Thick line: in the case of 24 time observations a day, broken line: 6 time observations, thin line: 3 time observations.

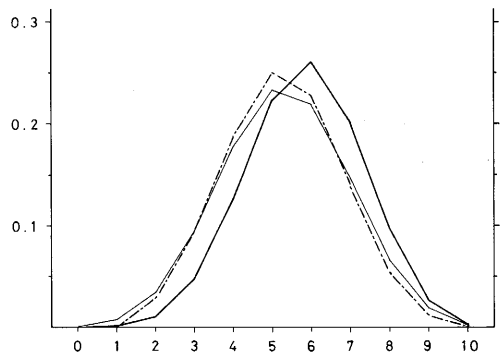

Fig. 2 Same as Fig. 1, except for 10 consecutive days.

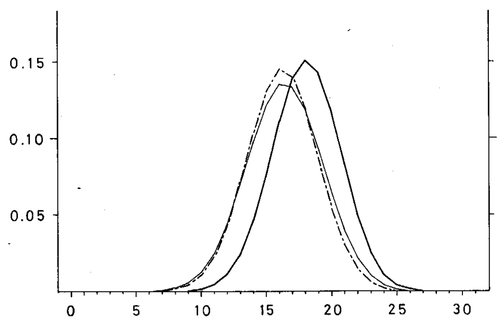

Fig. 3 Same as Fig. 1, except for 31 consecutive days.

Calculated and Actual Distributions of Days with Minus Deviation

Both calculated and actual distributions of days with minus deviation were shown in Table 3(a) and 3(b). Actual distribution of days with minus deviation in the case of 31 consecutive days was eliminated because of being scanty of data (Table 3(c)). Tests of goodness of fit were not significant at the 5 per cent level both in the cases of 5 and 10 consecutive days.

Table 3(a) Calculated and actual distributions of days with minus deviation in 5 consecutive days.

(A): in the case of 24 time observations a day, (B): 6 time observations,
(C): 3 time observations

	The number of days with minus deviation	Calculated distribution of days (days)	Actual distribution of days (days)
(A)	0	1.387	0
	1	11.626	11
	2	36.136	30
	3	51.637	61
	4	33.963	35
	5	8.265	6
(B)	0	3.116	5
	1	18.584	15
	2	42.653	39
	3	46.911	56
	4	24.731	23
	5	5.006	3
(C)	0	4.003	2
	1	19.598	21
	2	40.954	49
	3	45.576	38
	4	27.014	24
	5	6.840	9

Tests of goodness of fit χ^2 = 4.813 with 3 d.f., P = 0.10 – 0.25 for (A)
4.829 with 3 d.f., P = 0.10 – 0.25 for (B)
5.739 with 3 d.f., P = 0.10 – 0.25 for (C)

Table 3(b) Same as Table 3(a), except for 10 consecutive days.

	The number of days with minus deviation	Calculated distribution of days (days)	Actual distribution of days (days)
(A)	0	0.007	0
	1	0.104	0
	2	0.773	0
	3	3.278	0
	4	8.756	10
	5	15.394	17
	6	18.016	18
	7	13.876	15
	8	6.728	8
	9	1.856	1
	10	0.221	0
(B)	0	0.034	0
	1	0.388	0
	2	2.067	2
	3	6.406	8
	4	12.730	13
	5	16.986	17
	6	15.409	10
	7	9.391	14
	8	3.672	3
	9	0.836	1
	10	0.082	0
(C)	0	0.058	0
	1	0.569	0
	2	2.540	2
	3	6.964	12
	4	12.936	16
	5	16.987	11
	6	15.987	12
	7	10.651	8
	8	4.803	9
	9	1.329	3
	10	0.168	0

Tests of goodness of fit χ^2 = 5.452 with 8 d.f., P = 0.50 – 0.75 for (A)
5.224 with 8 d.f., P = 0.50 – 0.75 for (B)
14.811 with 8 d.f., P = 0.05 – 0.10 for (C)

Table 3(c) Probability distributions of days with minus deviation in 31 consecutive days.

(A): in the case of 24 time observations a day, (B): 6 time observations,
(C): 3 time observations

The number of days with minus deviation	(A) PROBABILITY	(B) PROBABILITY	(C) PROBABILITY
0	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000
6	0.0000	0.0001	0.0002
7	0.0000	0.0004	0.0007
8	0.0001	0.0015	0.0020
9	0.0004	0.0043	0.0054
10	0.0015	0.0109	0.0123
11	0.0044	0.0237	0.0249
12	0.0112	0.0447	0.0443
13	0.0246	0.0734	0.0698
14	0.0467	0.1052	0.0977
15	0.0772	0.1322	0.1218
16	0.1110	0.1455	0.1354
17	0.1389	0.1404	0.1341
18	0.1512	0.1185	0.1182
19	0.1428	0.0875	0.0927
20	0.1167	0.0562	0.0643
21	0.0821	0.0313	0.0394
22	0.0495	0.0150	0.0211
23	0.0253	0.0062	0.0099
24	0.0109	0.0021	0.0040
25	0.0039	0.0006	0.0014
26	0.0011	0.0001	0.0004
27	0.0003	0.0000	0.0001
28	0.0000	0.0000	0.0000
29	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000
31	0.0000	0.0000	0.0000

Probability of Total Deviating Amounts in the Cases of 5, 10 and 31 Consecutive Days

On the assumption, as mentioned before, that a relative frequency of occurrence of a plus (or minus) deviating amount in the actual sample of days with plus (or minus) deviations is probability of occurrence of the deviating amount of a day with plus (or minus) deviation, D (n, k, d) was calculated using relative frequencies of occurrence of deviating amounts in the actual sample (Table 4). There was a deviating amount 1.5°C in the case of 3 time observations a day. A relative frequency of it was 0.0027. This frequency was eliminated in calculation for reducing memory size of computer. Probability of occurrence of total deviating amounts calculated by the equation (2) were shown in Tables 5, 6 and 7. Tests of goodness of fit were not significant at the 5 per cent level.

Table 4 Relative frequencies of plus and minus deviating amounts (1/10°C) in the actual sample of days with plus and minus deviations

Numbers in brackets are actual numbers (days).
(A): in the case of 24 time observations a day, (B): 6 time observations,
(C): 3 time observations

DEVIATING AMOUNT (1/10°C)	(A) Relative frequency	(B) Relative frequency	(C) Relative frequency
11			0.0055 (2)
10			0.0082 (3)
9		0.0054 (2)	0.0165 (6)
8		0.0027 (1)	0.0275 (10)
7		0.0108 (4)	0.0467 (17)
6		0.0350 (13)	0.0742 (27)
5	0.0126 (4)	0.0755 (28)	0.0824 (30)
4	0.0252 (8)	0.0916 (34)	0.1017 (37)
3	0.0755 (24)	0.1509 (56)	0.1291 (47)
2	0.2201 (70)	0.2345 (87)	0.1896 (69)
1	0.4182 (133)	0.2534 (94)	0.2143 (78)
+0	0.2484 (79)	0.1402 (52)	0.1016 (37)
−0	0.2151 (97)	0.1629 (65)	0.0739 (30)
−1	0.3570 (161)	0.2481 (99)	0.2414 (98)
−2	0.2683 (121)	0.2607 (104)	0.1921 (78)
−3	0.1131 (51)	0.1353 (54)	0.1232 (50)
−4	0.0355 (16)	0.1003 (40)	0.1256 (51)
−5	0.0044 (2)	0.0426 (17)	0.0936 (38)
−6	0.0044 (2)	0.0276 (11)	0.0493 (20)
−7	0.0000 (0)	0.0100 (4)	0.0493 (20)
−8	0.0022 (1)	0.0025 (1)	0.0246 (10)
−9		0.0050 (2)	0.0074 (3)
−10		0.0050 (2)	0.0049 (2)
−11			0.0049 (2)
−12			0.0049 (2)
−13			0.0049 (2)

Table 5 Probability of total deviating amounts (1/10°C) in 5 consecutive days.

(A): in the case of 24 time observations a day

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0973		
1	0.0836	−1	0.1053
2	0.0668	−2	0.1058
3	0.0497	−3	0.0989
4	0.0345	−4	0.0859
5	0.0225	−5	0.0695
6	0.0140	−6	0.0524
7	0.0083	−7	0.0368
8	0.0048	−8	0.0243
9	0.0026	−9	0.0151
10	0.0014	−10	0.0088
11	0.0007	−11	0.0050
12	0.0003	−12	0.0027
13	0.0002	−13	0.0014
14	0.0001	−14	0.0007
15	0.0000	−15	0.0003
⋮	⋮	−16	0.0002
		−17	0.0001
		−18	0.0000
25	0.0000	⋮	⋮
		−40	0.0000

(B): 6 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0646		
1	0.0634	−1	0.0641
2	0.0605	−2	0.0619
3	0.0563	−3	0.0581
4	0.0509	−4	0.0532
5	0.0449	−5	0.0474
6	0.0385	−6	0.0411
7	0.0323	−7	0.0347
8	0.0264	−8	0.0286
9	0.0212	−9	0.0230
10	0.0166	−10	0.0180
11	0.0127	−11	0.0138
12	0.0096	−12	0.0103
13	0.0071	−13	0.0076
14	0.0052	−14	0.0054
15	0.0037	−15	0.0038
16	0.0026	−16	0.0026
17	0.0018	−17	0.0018
18	0.0012	−18	0.0012
19	0.0008	−19	0.0008
20	0.0005	−20	0.0005
21	0.0003	−21	0.0003
22	0.0002	−22	0.0002
23	0.0001	−23	0.0001
24	0.0001	−24	0.0001
25	0.0000	−25	0.0000
⋮	⋮	⋮	⋮
45	0.0000	−50	0.0000

(C) : 3 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0439		
1	0.0431	−1	0.0442
2	0.0418	−2	0.0439
3	0.0400	−3	0.0432
4	0.0378	−4	0.0419
5	0.0352	−5	0.0401
6	0.0325	−6	0.0379
7	0.0296	−7	0.0354
8	0.0266	−8	0.0326
9	0.0236	−9	0.0297
10	0.0208	−10	0.0267
11	0.0181	−11	0.0236
12	0.0155	−12	0.0207
13	0.0132	−13	0.0179
14	0.0111	−14	0.0152
15	0.0092	−15	0.0128
16	0.0076	−16	0.0106
17	0.0062	−17	0.0087
18	0.0050	−18	0.0071
19	0.0040	−19	0.0057
20	0.0031	−20	0.0045
21	0.0024	−21	0.0035
22	0.0019	−22	0.0027
23	0.0014	−23	0.0021
24	0.0011	−24	0.0016
25	0.0008	−25	0.0012
26	0.0006	−26	0.0009
27	0.0004	−27	0.0006
28	0.0003	−28	0.0005
29	0.0002	−29	0.0003
30	0.0001	−30	0.0002
31	0.0001	−31	0.0002
32	0.0001	−32	0.0001
33	0.0000	−33	0.0001
⋮	⋮	−34	0.0001
		−35	0.0000
55	0.0000	⋮	⋮
		−65	0.0000

Table 6 Same as Table 5, except for 10 consecutive days.
(A): in the case of 24 time observations a day

DEVIATING AMOUNT (1/10°C	PROBABILITY	DEVIATING AMOUNT (1/10°)	PROBABILITY
0	0.0629		
1	0.0552	−1	0.0691
2	0.0468	−2	0.0733
3	0.0383	−3	0.0750
4	0.0303	−4	0.0740
5	0.0231	−5	0.0705
6	0.0171	−6	0.0648
7	0.0122	−7	0.0575
8	0.0084	−8	0.0492
9	0.0056	−9	0.0407
10	0.0036	−10	0.0325
11	0.0023	−11	0.0251
12	0.0014	−12	0.0187
13	0.0008	−13	0.0136
14	0.0005	−14	0.0095
15	0.0003	−15	0.0065
16	0.0001	−16	0.0043
17	0.0001	−17	0.0027
18	0.0000	−18	0.0017
⋮	⋮	−19	0.0011
		−20	0.0006
		−21	0.0004
50	0.0000	−22	0.0002
		−23	0.0001
		−24	0.0001
		−25	0.0000
		⋮	⋮
		−80	0.0000

(B): 6 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0455		
1	0.0450	−1	0.0454
2	0.0439	−2	0.0447
3	0.0423	−3	0.0434
4	0.0402	−4	0.0416
5	0.0378	−5	0.0394
6	0.0350	−6	0.0368
7	0.0320	−7	0.0340
8	0.0289	−8	0.0309
9	0.0257	−9	0.0278
10	0.0227	−10	0.0246
11	0.0197	−11	0.0216
12	0.0169	−12	0.0187
13	0.0143	−13	0.0159
14	0.0120	−14	0.0134
15	0.0099	−15	0.0112
16	0.0081	−16	0.0092
17	0.0065	−17	0.0075
18	0.0052	−18	0.0060
19	0.0041	−19	0.0048
20	0.0032	−20	0.0038
21	0.0024	−21	0.0029
22	0.0018	−22	0.0022
23	0.0014	−23	0.0017
24	0.0010	−24	0.0013
25	0.0007	−25	0.0009
26	0.0005	−26	0.0007
27	0.0004	−27	0.0005
28	0.0003	−28	0.0004
29	0.0002	−29	0.0003
30	0.0001	−30	0.0002
31	0.0001	−31	0.0001
32	0.0001	−32	0.0001
33	0.0000	−33	0.0001
⋮	⋮	−34	0.0000
90	0.0000	⋮	⋮
		−100	0.0000

(c): 3 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0307		
1	0.0302	−1	0.0309
2	0.0296	−2	0.0310
3	0.0288	−3	0.0309
4	0.0278	−4	0.0306
5	0.0268	−5	0.0301
6	0.0256	−6	0.0294
7	0.0243	−7	0.0286
8	0.0229	−8	0.0276
9	0.0215	−9	0.0265
10	0.0200	−10	0.0253
11	0.0186	−11	0.0240
12	0.0171	−12	0.0226
13	0.0157	−13	0.0211
14	0.0142	−14	0.0197
15	0.0129	−15	0.0182
16	0.0116	−16	0.0167
17	0.0103	−17	0.0152
18	0.0092	−18	0.0138
19	0.0081	−19	0.0125
20	0.0071	−20	0.0112
21	0.0062	−21	0.0099
22	0.0054	−22	0.0088
23	0.0046	−23	0.0077
24	0.0039	−24	0.0067
25	0.0033	−25	0.0059
26	0.0028	−26	0.0050
27	0.0024	−27	0.0043
28	0.0020	−28	0.0037
29	0.0016	−29	0.0031
30	0.0013	−30	0.0026
31	0.0011	−31	0.0022
32	0.0009	−32	0.0018
33	0.0007	−33	0.0015
34	0.0006	−34	0.0012
35	0.0005	−35	0.0010
36	0.0004	−36	0.0008
37	0.0003	−37	0.0007
38	0.0002	−38	0.0005
39	0.0002	−39	0.0004
40	0.0001	−40	0.0003
41	0.0001	−41	0.0003
42	0.0001	−42	0.0002
43	0.0001	−43	0.0002
44	0.0000	−44	0.0001
⋮	⋮	−45	0.0001
		−46	0.0001
		−47	0.0001
110	0.0000	−48	0.0000
		⋮	⋮
		−130	0.0000

Table 7 Same as Table 5, except for 31 consecutive days.

(A): in the case of 24 time observations a day

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0247		
1	0.0220	−1	0.0274
2	0.0193	−2	0.0301
3	0.0168	−3	0.0327
4	0.0145	−4	0.0352
5	0.0123	−5	0.0373
6	0.0103	−6	0.0392
7	0.0086	−7	0.0407
8	0.0071	−8	0.0417
9	0.0057	−9	0.0423
10	0.0046	−10	0.0425
11	0.0037	−11	0.0421
12	0.0029	−12	0.0413
13	0.0022	−13	0.0400
14	0.0017	−14	0.0383
15	0.0013	−15	0.0363
16	0.0010	−16	0.0340
17	0.0007	−17	0.0315
18	0.0005	−18	0.0289
19	0.0004	−19	0.0262
20	0.0003	−20	0.0234
21	0.0002	−21	0.0207
22	0.0001	−22	0.0182
23	0.0001	−23	0.0157
24	0.0001	−24	0.0135
25	0.0000	−25	0.0114
⋮	⋮	−26	0.0096
		−27	0.0079
		−28	0.0065
155	0.0000	−29	0.0053
		−30	0.0042
		−31	0.0034
		−32	0.0026
		−33	0.0021
		−34	0.0016
		−35	0.0012
		−36	0.0009
		−37	0.0007
		−38	0.0005
		−39	0.0004
		−40	0.0003
		−41	0.0002
		−42	0.0001
		−43	0.0001
		−44	0.0001
		−45	0.0000
		⋮	⋮
		−248	0.0000

(B): 6 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0257		
1	0.0255	−1	0.0258
2	0.0253	−2	0.0257
3	0.0249	−3	0.0255
4	0.0244	−4	0.0253
5	0.0239	−5	0.0249
6	0.0232	−6	0.0245
7	0.0225	−7	0.0239
8	0.0217	−8	0.0233
9	0.0209	−9	0.0225
10	0.0200	−10	0.0218
11	0.0190	−11	0.0209
12	0.0181	−12	0.0200
13	0.0171	−13	0.0191
14	0.0161	−14	0.0181
15	0.0151	−15	0.0171
16	0.0141	−16	0.0161
17	0.0131	−17	0.0151
18	0.0121	−18	0.0141
19	0.0112	−19	0.0131
20	0.0102	−20	0.0121
21	0.0094	−21	0.0112
22	0.0085	−22	0.0103
23	0.0077	−23	0.0094
24	0.0070	−24	0.0086
25	0.0063	−25	0.0078
26	0.0056	−26	0.0070
27	0.0050	−27	0.0063
28	0.0044	−28	0.0057
29	0.0039	−29	0.0050
30	0.0035	−30	0.0045
31	0.0030	−31	0.0040
32	0.0027	−32	0.0035
33	0.0023	−33	0.0031
34	0.0020	−34	0.0027
35	0.0017	−35	0.0023
36	0.0015	−36	0.0020
37	0.0013	−37	0.0018
38	0.0011	−38	0.0015
39	0.0009	−39	0.0013
40	0.0008	−40	0.0011
41	0.0007	−41	0.0009
42	0.0005	−42	0.0008
43	0.0005	−43	0.0007
44	0.0004	−44	0.0006
45	0.0003	−45	0.0005
46	0.0003	−46	0.0004
47	0.0002	−47	0.0003
48	0.0002	−48	0.0003
49	0.0001	−49	0.0002
50	0.0001	−50	0.0002
51	0.0001	−51	0.0001
52	0.0001	−52	0.0001

(Continued)

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
53	0.0001	−53	0.0001
54	0.0000	−54	0.0001
⋮	⋮	−55	0.0001
		−56	0.0000
279	0.0000	⋮	⋮
		−310	0.0000

(c): 3 time observations

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
0	0.0166		
1	0.0164	−1	0.0167
2	0.0161	−2	0.0169
3	0.0159	−3	0.0170
4	0.0156	−4	0.0171
5	0.0153	−5	0.0171
6	0.0149	−6	0.0171
7	0.0146	−7	0.0171
8	0.0142	−8	0.0170
9	0.0138	−9	0.0169
10	0.0134	−10	0.0168
11	0.0130	−11	0.0167
12	0.0125	−12	0.0165
13	0.0121	−13	0.0162
14	0.0116	−14	0.0160
15	0.0112	−15	0.0157
16	0.0107	−16	0.0154
17	0.0102	−17	0.0151
18	0.0098	−18	0.0147
19	0.0093	−19	0.0144
20	0.0089	−20	0.0140
21	0.0084	−21	0.0136
22	0.0080	−22	0.0131
23	0.0075	−23	0.0127
24	0.0071	−24	0.0123
25	0.0067	−25	0.0118
26	0.0063	−26	0.0114
27	0.0059	−27	0.0109
28	0.0055	−28	0.0104
29	0.0052	−29	0.0100
30	0.0048	−30	0.0095
31	0.0045	−31	0.0090
32	0.0042	−32	0.0086
33	0.0039	−33	0.0081
34	0.0036	−34	0.0077

(Continued)

DEVIATING AMOUNT (1/10°C)	PROBABILITY	DEVIATING AMOUNT (1/10°C)	PROBABILITY
35	0.0033	−35	0.0073
36	0.0030	−36	0.0069
37	0.0028	−37	0.0064
38	0.0026	−38	0.0060
39	0.0023	−39	0.0057
40	0.0021	−40	0.0053
41	0.0020	−41	0.0049
42	0.0018	−42	0.0046
43	0.0016	−43	0.0043
44	0.0015	−44	0.0040
45	0.0013	−45	0.0037
46	0.0012	−46	0.0034
47	0.0011	−47	0.0031
48	0.0010	−48	0.0029
49	0.0009	−49	0.0026
50	0.0008	−50	0.0024
51	0.0007	−51	0.0022
52	0.0006	−52	0.0020
53	0.0006	−53	0.0018
54	0.0005	−54	0.0017
55	0.0004	−55	0.0015
56	0.0004	−56	0.0014
57	0.0003	−57	0.0012
58	0.0003	−58	0.0011
59	0.0003	−59	0.0010
60	0.0002	−60	0.0009
61	0.0002	−61	0.0008
62	0.0002	−62	0.0007
63	0.0002	−63	0.0006
64	0.0001	−64	0.0006
65	0.0001	−65	0.0005
66	0.0001	−66	0.0005
67	0.0001	−67	0.0004
68	0.0001	−68	0.0004
69	0.0001	−69	0.0003
70	0.0001	−70	0.0003
71	0.0000	−71	0.0002
⋮	⋮	−72	0.0002
⋮	⋮	−73	0.0002
⋮	⋮	−74	0.0002
341	0.0000	−75	0.0001
		−76	0.0001
		−77	0.0001
		−78	0.0001
		−79	0.0001
		−80	0.0001
		−81	0.0001
		−82	0.0001
		−83	0.0000
		⋮	⋮
		−403	0.0000

Calculated and Actual Distributions of Deviating Amounts in Mean Temperatures in 5, 10 and 31 Consecutive Days

In the cases of 5 and 10 consecutive days, actual distributions of deviating amounts in mean temperature agreed well with the calculated distributions (Figs. 4 and 5). An actual distribution of deviating amounts in mean temperature in 31 consecutive days was eliminated because of being scanty of data (Fig. 6).

It became clear from Figs. 4 and 5 that in the case of 3 time observations probability of plus and minus deviations were somewhat larger and that of no deviation smaller. It also became clear that the shorter the period of days to be averaged was, the smaller probability of no deviation was and at the same time the larger probability of plus and minus deviations

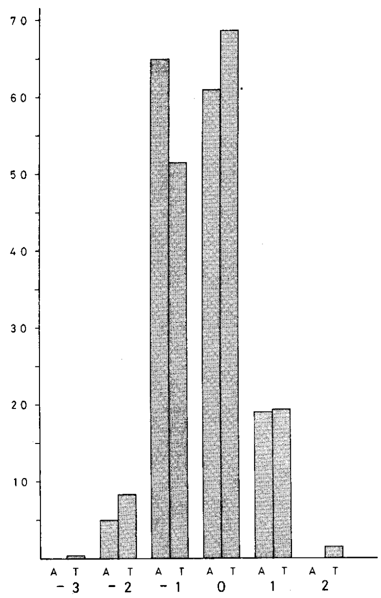

Fig. 4(a) Calculated and actual distributions of deviating amounts in mean temperatures of 5 consecutive days in the case of 24 time observations a day.

Ordinate: the number of days, abscissa: deviating amounts in mean temperatures (1/10°C). A: actual, T: theoretical.

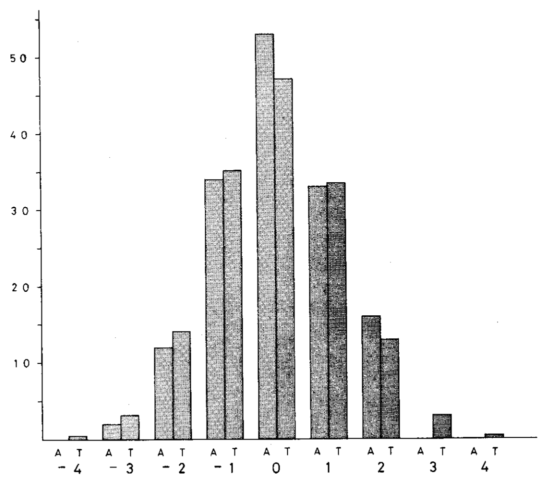

Fig. 4(b) Same as Fig. 4(a), except for in the case of 6 time observations a day.

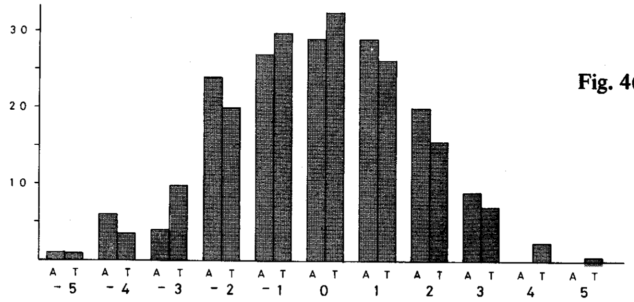

Fig. 4(c) Same as Fig. 4(a), except for in the case of 3 time observations a day.

were. On the other hand, in the case of monthly mean temperature, it was clear that probability of no deviation was very large and any probability of plus and minus deviations were extremely small (Fig. 6).

In the case of 3 time observations probability of minus deviation were somewhat larger than that of plus deviation in each kind of mean temperature.

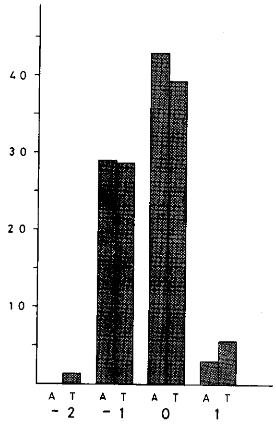

Fig. 5(a) Calculated and actual distributions of deviating amount in mean temperatures of 10 consecutive days in the case of 24 time observations a day.

Ordinate: the number of days, abscissa: deviating amounts in mean temperatures (1/10°C). A: actual, T: theoretical.

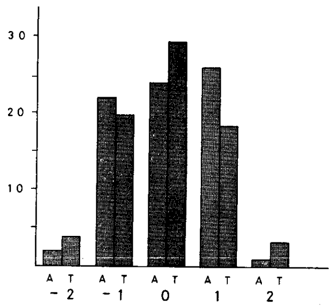

Fig. 5(b) Same as Fig. 5(a), except for in the case of 6 time observations a day.

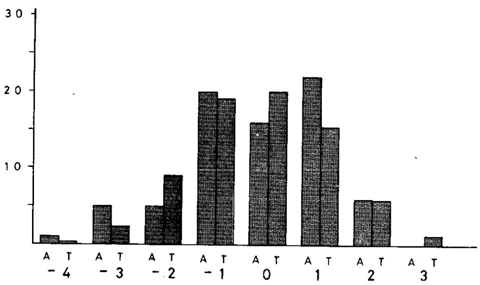

Fig. 5(c) Same as Fig. 5(a), except for in the case of 3 time observations a day.

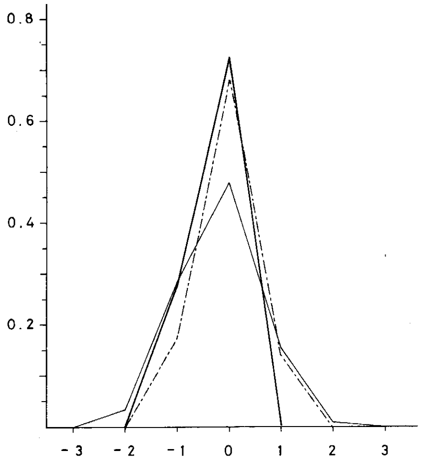

Fig. 6 Probability of deviating amounts in mean temperatures of 31 consecutive days.

Ordinate: Probability, abscissa: deviating amounts in mean temperatures (1/10°C). Thick line: in the case of 24 time observations a day, broken line: 6 time observations, thin line: 3 time observations.

DISCUSSIONS AND CONCLUSIONS

In this research it was found that deviations of ${}_1T_{24}$, ${}_1T_6$ and ${}_1T_3$ from ${}_1T_8$ occurred according to the Markov chain model. Some statistical analysis of meteorological data, such as probability of spells, can be done by applications of a Markov chain model. It should be, however, understood that these analyses do not prove any physical basis for the model.

On the assumption that a relative frequency of occurrence of a plus (or minus) deviating amount in the actual sample of days with plus (or minus) deviations is probability of occurrence of the deviating amount of a day with plus (or minus) deviation, probability of deviating amounts of mean temperatures were calculated. From good agreement of the calculated probability with the actual samples, the assumption may be said to be acceptable.

As a daily mean temperature is calculated using fixed time observed temperatures, diurnal variation of temperature may determine a day with plus (or minus) deviation and a deviating amount of a day. Diurnal variation of temperature at a region is dependent on factors such as daily weather, latitude and longitude of the region, season and so on. Therefore, parameters of a model which reflect features of the factors of a region, may be a kind of indeces to classify climatic regions, though this climatic classification dose not prove any physical basis for the classification.

As Maebashi meteorological station did not move during the period of which data were processed, changes of environment around the station caused by displacement were excluded. However, the so-called urban climatic influence was not excluded. As the influence varies to large extent according to general atmospheric conditions, it may be difficult to evaluate urban climatic influence on diurnal variation of temperature. It may be a problem to be solved.

In researches such as of climatic change, which deal with long-term climatic data series, it is a problem to regard how much deviating amounts as significant. From the results obtained in this research, it may be said that the change in monthly mean temperature caused by changes of the calculating formulae may be almost ignored. On the contrary, in the cases of five- and ten-day mean temperatures it may require care to handle the changes within about ±0.5°C.

It was found in this research that occurrence of a day with plus (or minus) deviation followed the Markov chain model, and that probability of no deviation in mean temperature was the largest and furthermore probability of minus deviations were somewhat larger than that of plus deviations. It was also found that the less times of observations a day was or the shorter the period of days to be averaged was, the smaller probability of no deviation was and at the same time the larger probability of plus and minus deviation was.

ACKNOWLEDGEMENTS

This paper is dedicated to Prof. Taiji Yazawa on the occasion of his retirement from Tokyo Metropolitan Univ., who has led the author to essentials of geography, especially of climatology. The author is indebted to Prof. Ikuo Maejima of Tokyo Metropolitan Univ. for his critical reading of the present paper.

REFERENCES CITED

Anderson, T.W. & Goodman, L.A. (1957): Statistical inference about Markov chains. *Ann. Math. Stat.* 28, 89–110.

Feller, W. (1957): *An Introduction to Probability Theory and its Applications. Vol. 1.* 2nd ed. John Wiley & Sons, Inc. New York.

Gabriel, K.R. & Neumann, J. (1962): A Markov chain model for daily rainfall occurrence at Tel Aviv. *OJRMS.* 88, 90–95.

Katz, W. (1974): Computing probabilities associated with the Markov chain model for precipitation. *Jour. Appl. Met.* 13, 953–954.

Statistical Section, JMA (1961): Difference among the daily mean values of meteorological elements averaged by the different methods. *Jour. Met. Res.* 13, 864–880, (in Japanese).

Thom, H.C.S. (1966): Some methods of climatological analysis. *WMO, Technical Note* No. 81, 53p.