Doctoral Thesis

# Planning and Management of Worker Assignment in a Line Production System

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# Abstract

Line production systems are widely used as part of the production process for the mass production of automobiles and home appliances. Line production systems are operated with a fixed cycle time, and line balancing is performed to equalize the standard working time of each process. After the line balancing, worker assignment is carried out. However, since the capability to perform work is different for each worker, work may not be completed on time, idle time may occur, and a delay in one process may cause a delay in the following process. A problem that deals with the loss of work time over multiple processes under the constraint of a fixed time cycle as the total expected cost is called a limited-cycle problem with multiple periods. In this problem, a model designed to prevent an adverse effect on a subsequent process in the event of a work delay in one process by increasing the number of relief workers is called a reset model, whereas a model in which a work delay in any process has a direct impact on the subsequent process is called a non-reset model.

Previous studies proposed rules of worker assignment that provide a formula for the total expected cost in the reset model and minimize this total expected cost. The conventional methods assume that each worker has one of two levels of work capability, specifically labeling each worker as either proficient or inefficient. A proficient worker can complete work in all the processes according to a standard schedule, while an inefficient worker needs more time than the standard scheduled time to complete the work of all the processes. In fact, every worker has their own strong points and weak points according to the tasks assigned. Therefore, to match actual real-world conditions, each worker's level (proficiency or inefficiency) should be set for each process. However, no method of worker assignment that minimizes the total expected cost for such a model has yet been developed. In view of this, the present thesis proposes a method of worker assignment which expands the assumptions of the previous studies and minimizes the total expected cost while assuming the level of proficiency or inefficiency of each worker is set for each process. Furthermore, since the actual working time of each process changes according to each worker's degree of skill, the replacement of workers, etc., this thesis proposes a method of automated work analysis for the purpose of measuring the actual working time of each process and using the results in a re-evaluation of the work capability of each worker and a re-planning of the worker assignment. This

thesis consists of five chapters.

Chapter 1 reviews the background of the present research and prior relevant research on assembly line balancing problems. The objectives of the present research are then stated, and the thesis statement is given.

Chapter 2 states the improvement approach in a line production system. Next, the matter of theory and practice in worker assignment is discussed. Prior relevant research, including research on worker assignment and assessment of work performance based on work analysis, is then reviewed. Additionally, the significance of the present research is given.

Chapter 3 discusses the problem of optimal assignment in the planning of worker assignment. Firstly, a method is proposed for calculating the total expected cost using a stochastic model when workers have different processing capabilities for different processes. Secondly, a simple case is discussed where each process allows two levels (proficiency and inefficiency) of workers' processing capabilities and a method is presented for calculating the total expected cost applicable to this case. Subsequently, some theorems are proved concerning the optimal assignment of workers for the case that an inefficient worker is assigned to only one task (i.e., proficient workers can be assigned to all the remaining tasks). The theoretical optimal assignment of workers is proved by a recursive method. According to the optimal assignment, inefficient workers should be placed either at the starting point or at the ending point of the process, while proficient workers should be positioned at the remaining points of the process. The validity of the theorem is confirmed through numerical experiments. Lastly, the differences between the estimated values and the measured values are examined by measuring the working time of each process and a method is presented for worker assignment based on the optimal assignment rules.

Chapter 4 describes a work analysis for assessing workers' performance. An automated work analysis system that makes use of an ultrasonic network, a tracking camera, and an acceleration sensor is developed. The tracking camera and ultrasonic network are used to determine the position and moving speed. Firstly, position measurement using an ultrasonic sensor network is explained and formalized. Then, a new process for connecting a flow line measured by the ultrasonic sensor and a flow line measured by the tracking camera is described. Subsequently, a method of motion estimation is discussed. Using extensive cross-validation, the most accurate estimation model is chosen based on position measurement data (i.e., position and moving speed), and acceleration data. Furthermore, an experiment is conducted to test the measurement accuracy of the proposed method. The results of the motion estimation are found to be more than 90% consistent with those of a video analysis. Finally, an assessment of workers' performance based on the work analysis is discussed.

Chapter 5 summarizes the results of the present research and proposes areas for further research. The expected use of the proposed worker assignment method is discussed.

# Contents

# List of Figures

# List of Tables

# Chapter 1 Introduction

This chapter presents the background of this study, a review of prior relevant literature, and the organization of the thesis. In a related literature review, the line balancing problem and methods to improve line balancing are introduced, as well as the current challenges in researching the line balancing problem. The objectives of the study and the organization of the thesis are then stated.

## 1.1 Research background

From the beginning of Henry Ford to today's era of Industry 4.0, the assembly line production system occupies an important position in industrial production, and there is an increasing interest in how to improve production efficiency in production systems composed of existing assembly lines[1][2]. Due to increasing scarce external energy resources and rising prices of raw materials and labor costs[3], productivity needs to be improved by applying management methods and production techniques that maximize the use of people, materials, money, and information to produce products and meet specified quality, cost, and delivery dates[4]. On-site management is widely used in production sites. It is a continuous optimization process to analyze and solve the problems found in the production process[5]. Through a method called the PDCA cycle (Plan-Do-Check-Action cycle), quality work is planned, executed, checked, and acted upon to ensure the target of reliability is achieved and to promote continuous quality improvement[6]. In addition, as an important element of on-site management, line balancing is used for designing production processes and improving production efficiency.

Improving the balance of each process in the assembly line can reduce the time difference between processes and shorten the waiting time for product production and maintain continuous production. It can also prevent suspension and enhance the efficiency of product production[7].

In production control activities, line balancing is often taken into consideration in the production process planning stage[8]. Proper and coordinated production planning and its adjustment to changing conditions can have a positive impact on productivity and increase productivity and minimize costs[9].

In the design of the balance of the assembly line, the difference in process time between each process should not be too great when the workers' standardized operation is in place, and consistency should be maintained as much as possible. Consistency is achieved by standardizing operations in the field. The optimal working procedure is used as the standard operation. Under suitable operating conditions, the time required to complete the standard operation with the optimal operation method and at the normal speed of ordinary workers is regarded as standard working time[10][11][12]. Standard working hours and standard operations can be used as a basis for planning, while the application of work standardization enables all personnel involved in the work to plan, implement and assess the work results objectively and accurately, thus effectively improving productivity[13].

With the continuous development of science and technology, the reliability of machines on the assembly line has increased significantly, but workers are still an important part of the production process. Workers still perform most operations in the manufacturing industry[14]. In a worker-operated assembly line, the variation of worker efficiency is highly dependent on the processing capability of the worker, so when the target working time (cycle time) of each process is fixed, the actual working time of the worker may change probabilistically. If a target working time (cycle time) is defined for each process and used as a constraint cycle for each process, then the actual working time of workers will change probabilistically. In this case, idle time occurs when the actual working time is less than the target working time, and conversely, delay occurs when the actual working time is more than the target working time. If there is a delay in a process, it will affect the subsequent processes, causing delays in delivery and increased costs. This means that each process is subject to a time constraint as a period in a limited-cycle model with multiple periods. It shows a situation where there is a risk associated with a delivery date constraint in a period, and if this risk extends to multiple periods and the constraint is not satisfied, the whole system is affected. In such a limited-cycle model with multiple periods, hiring more workers would be used to eliminate the risk of not being able to complete production during the delivery period, thus turning the impact on time delays into an impact on costs. At this time, taking into account the differences in workers' processing capabilities, a reasonable assignment of workers is the way to solve such problems. The way workers are assigned in the system greatly affects system performance[15]. In addition, differences in objective physiological conditions and subjective emotional-psychological factors[16] among assembly line workers can cause different workers to have different working time fluctuations for the same task. The difference between actual time and standard working times is caused by the difference in workers' capability. Various factors generally need to be considered when planning the assignment of workers.

A reasonable worker assignment can present good production synergy among workstations in production. An unreasonable worker assignment requires worker readjustment, which involves the consideration of worker assignment optimization to avoid the accumulation of tasks at some workstations and the loss of work hours at some workstations. The focus in the management of worker assignment is first to assess their capabilities effectively and then to place them in tasks that fit their capabilities in the new mode. In the production system, the performance of workers is assessed based on the study of their motions and working times to detect the degree of understanding, execution, and effectiveness of standardized operations. Performance assessment results are used to make timely adjustments to optimize the assembly line.

## 1.2 Overview of line balancing problem

### 1.2.1 The problem of line balancing

Assembly line is a production method that is widely used in production systems. The assembly workstations are arranged in the sequence of the product's assembly process route, and the object is to be processed on the line in a specific order, from the basic parts, with some parts gradually attached at each workstation, to finally form the finished product[17].

Bryton first mentioned the concept of the assembly line balancing problem in 1954. He proposed to exchange the task elements in each workstation until the task time of each workstation converged to the same time value, and the assembly line reached a certain balance state[18].

Line balancing is the technical means of averaging all processes and adjusting the working load so that the working time is as similar as possible, to eliminate the excessive accumulation of tasks in one process while other processes are left idle. It is the most important method in production flow design and operation standardization[19]. To maximize the performance of the assembly line, the production of each workstation in the line must be balanced, which means that the assembly line is in balance.

To effectively evaluate the effectiveness of the assembly line before and after balancing, several evaluation indicators need to be applied. The most common of these is the line efficiency rate. Line efficiency, which is the primary indicator of economic performance, reflects positive achievement in line utilization[19] (Eq. (1.2.1)).

$$Line \quad efficiency = \frac{\begin{array}{c}Total\ time\ spent\\ on\ work\ elements\end{array}}{\begin{array}{c}Number \quad of \quad workstations\\ \times Maximum\ cycle\ time\end{array}} \times 100\% \qquad (1.2.1)$$

A good line efficiency rate represents low production cost, high productivity, and a high input-output ratio. Optimizing the balance of the assembly line helps to improve the utilization rate of workers and equipment and enhance production efficiency. Line balancing is an important theoretical basis for production managers to prepare production tasks and worker assignments, as well as an important indicator for actual production management.

### 1.2.2 Related research on methods for improving line balancing

Research on assembly line balancing methods can be broadly classified into three categories: optimization, heuristic, and industrial engineering. Optimization is based on a complete understanding of the assembly line process and on establishing mathematical models to find the optimal solution to the assembly line balancing problem. Salveson first proposed a linear programming model for the assembly line balancing problem using the mathematical analytical method in 1955 and studied the process of finding the optimal solution[20]. Jackson proposed an enumeration method to solve the assembly line balancing problem by listing the combinations of assignments on the assembly line and finding the optimal one from these combinations[21]. Bowman eh proposed to use a 0-1 linear programming model to solve the problem[22]. Dynamic programming methods[23] and network modeling methods[24] are also used to solve assembly line balancing problem. With the development of computer technology, the computational speed of solving mathematical models has been greatly enhanced[25], thus making the mathematical model method commonly used.

Heuristic can effectively avoid complex mathematical models, as well as can reduce the large amount of model data processing that exists later in the relative mathematical models. Kilbridge and Wester proposed a heuristic algorithm that uses the priority relationship graph as a constraint for solving the assembly line balancing problem[26].

Leu used a balancing rate as an objective function and applied a genetic algorithm for the first time to solve a single deterministic balancing problem[27]. The complexity of the programming code for the heuristic algorithm makes it difficult for many production site managers to understand the heuristic.

Industrial Engineering is used by relevant managers on the assembly line site[28]. Taylor used the data as a basis for his study of workers' work practices[10]. Gilbreth took the movements of the workers and made a film of them, proposing a more nuanced study of the motions using video analysis[29]. IE Association of Japan defines industrial engineering as “IE is concerned with the design, improvement, and establishment of integrated systems of people, goods, and equipment, and it uses the principles and methods of engineering analysis and design, as well as mathematics, physics, and the social sciences, to clarify, predict, and evaluate the results obtained from such systems. Utilizes specialized knowledge and skills and techniques of the mathematical, physical, and social sciences.”[30]. In production sites, production managers usually manage with their own experience and management level, which to a certain extent can limit the practical application of industrial engineering methods.

With the gradual research on the assembly line balancing problem, the study of optimization models is also developing towards optimization methods such as shortest path, integer programming, and dynamic programming[31][32][33][33][34]. Research on heuristic algorithms is also continuing[35][36][37][38]. Industrial engineering also has the trend of computer applications such as Enterprise Resources Planning（ERP） and Material Requirement Planning（MRP） to assist in the work.

### 1.2.3 Challenges in line balancing problem

The general discussion of line balancing is only about how to put work into workstations but often ignores how to put workers into workstations and still maintain the previous plan to achieve line balancing. When the production line is running by placing workers into it, it is likely that the expected line balancing will not be achieved due to worker factors, which will cause unintended losses. Since each worker's capability is actually different, if the workers are assigned according to the previously planned production line balance, there will be some workers who will not be able to carry out as planned, and the resulting idle time and delay time will affect the total time of the whole production line. As a result, an already balanced production line becomes unbalanced by ignoring individual worker efficiency differences (different processing capabilities for tasks).

In fact, workers have objective physiological conditions and subjective emotional psychology, and there are large differences among individual workers, such as emotional state, health condition, and capability level, which can have an impact on the balance of the assembly line. Therefore, the issue of optimizing the assignment of workers in the assembly line while considering the differences in worker efficiency is worthy of attention.

In the line production system, when workers repeatedly complete the same task, knowledge, as well as work experience, is accumulated, thus becoming more and more proficient, the working time required to complete an equal amount of the same task will gradually decrease, that is, there is the phenomenon that the time consumed to complete the product decreases with the increase in the number of work repetitions[39]. Małachowski and Korytkowski developed a model that relates work capability to worker performance based on a measure of the number of repetitive tasks performed[40]. Workers also have proficiency in their work capabilities, which affects the time to complete tasks, and they can improve their skills through repetitive work. Therefore, the learning capability of workers, the growth pattern of proficiency, and how to assign workers to tasks are worth studying in the context of worker assignment. Conversely, extensive manual work increases worker fatigue[41]. Work fatigue is the discomfort that gradually appears in the process of manual work, which is a state of the obvious decrease in work capability and a complex expression of working physiology and psychology, and is one of the important reasons for accidents and the decrease in production efficiency[42].

Proficiency and fatigue can make the efficiency of workers deviate from the plan, resulting in failure to perform properly according to the planning of worker assignment. That is to say, even if worker assignments are optimized through line balancing, the work capability of workers affected by proficiency and fatigue can change, making an otherwise balanced line unbalanced. At this point, it is crucial to assign workers according to their work capabilities and to get a realistic view of the work being performed in each process.

## 1.3 Research objectives

In this research, planning and management of worker assignment in a line production system are discussed. Concerning worker assignment, the planning of worker assignment and reassignment based on a theoretical model, work analysis in practice, and the assessment of the gap between the planned and actual work capability, and the improvement (such as work guidance, work improvement, and work standardization) will be continued. It will make the method of worker assignment more useful in practice. It is a continuous iteration of the theory and practice of worker assignment from the perspective of total expected cost minimization or productivity improvement in the management of the production line. This thesis focuses on two issues of the assignment of workers: the problem of optimal assignment in theory and a work analysis automation method in practice. In the planning of worker assignment, the purpose is to propose various optimal assignment rules applicable to the design of production systems to cope with different production conditions, taking into account the variability of workers' processing capabilities. The work analysis system measures the working time necessary for assessing work performance. When the working time changes, the theoretical model is returned and workers are reassigned.

The planning of worker assignment can be viewed as a theoretical model to find the worker assignment that minimizes the total expected cost by taking into account the quantified work capability of each worker under the work assignment equalized by line balancing. Line production systems are operated with a fixed cycle time, and line balancing is performed to equalize the standard working time of each process. After the line balancing, worker assignment is carried out. However, since the capability to perform work is different for each worker, work may not be completed on time, idle time may occur, and a delay in one process may cause a delay in the following process. A problem that deals with the loss of work time over multiple processes under the constraint of a fixed time cycle as the total expected cost is called a limited-cycle problem with multiple periods. In this problem, a model designed to prevent an adverse effect on a subsequent process in the event of a work delay in one process by increasing the number of relief workers is called a reset model. In the reset model, this thesis concerns the fact that work capability differs among workers and the difference in the work capability of the same worker for different tasks. The previous studies of this model assume that each worker has the same capability for all the processes. In fact, every worker has their strong points and weak points according to the tasks assigned. Therefore, it is suited to the actual circumstances if each worker's level (such as proficiency or inefficiency) is set for each process. However, no method of worker assignment that minimizes the total expected cost under these prerequisites has been developed yet. In view of this, this thesis aims to propose a formula for calculating the total expected cost and rules of worker assignment, which expands the assumptions of the previous studies and minimizes the total expected cost under the prerequisites where the level of proficiency or inefficiency of each worker is set for each process. Moreover, a method of applying the optimal rules to worker assignment is considered.

The purpose of the work analysis system is to assist in going back and forth between worker assignment in the theoretical model and in practice. Since work analysis is essential to turn the theory and practice of worker assignment, the idea is to develop a system that could perform work analysis with significantly less time and labor than video analysis. The automated work analysis system makes use of an ultrasonic network, a tracking camera, and an acceleration sensor. After explaining and formalizing the method of position measurement using the ultrasonic sensor network, a new process is proposed to connect the flow line measured by the ultrasonic sensor and the flow line measured by the tracking camera. Position and moving speed are obtained by using the ultrasonic network and the tracking camera. The most accurate estimation model is selected after extensive cross-validation based on position measurement data (i.e., position and moving speed), and the wavelet transforms coefficients of acceleration. An experiment is conducted to test the measurement accuracy of the proposed method. It is hoped that the system will achieve more than 90% consistency with the results of conventional video analysis. In addition, the assessment of workers' performance based on work analysis is discussed. By developing the work analysis system, the working time necessary for assessing work performance is measured. Furthermore, it will be possible to go back and forth between the theory and practice of worker assignment and utilize the method of worker assignment.

## 1.4 Thesis statement

This thesis discusses the planning and managing worker assignment in a line production system, aimed at making timely adjustments to optimize worker assignment.

Chapter 1 reviews the available research on the problem of line balancing. The objectives of the current study are then stated, and the thesis statement is given.

Chapter 2 states the improvement approach in a line production system. Next, the matter of theory and practice in worker assignment is discussed. Prior relevant research, including research on worker assignment and assessment of work performance based on work analysis, is then reviewed. Additionally, the significance of the present research is given.

Chapter 3 reviews the reset limited-cycle model with multiple periods and explains different workers' capabilities for each process. Focused on the differences in workers' capability to process the tasks, a method for calculating the total expected cost where the workers have different processing capabilities for different processes is presented. Next, the case where there are two types of processing capabilities (proficient or inefficient) of a worker for each process is discussed. A method for calculating the total expected cost applicable to this case is presented. Then, some theorems of the rule of the optimal worker assignment for special cases are described in relation to two special cases. In one of the cases, workers can be assigned to the tasks they are proficient at performing. In the other case, an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all remaining tasks. Additionally, a method for worker assignment based on the optimal assignment rules is discussed.

Chapter 4 describes proposes an automated work analysis system that makes use of an ultrasonic network, a tracking camera, and an acceleration sensor to access workers' performance. Position measurement and motion analysis are the two components of the work analysis approach. Firstly, the method of position measurement using an ultrasonic sensor network is explained and formalized. Then, a new process for connecting a flow line measured by the ultrasonic sensor and a flow line measured by the tracking camera is described. After that, the method of motion estimation is discussed. Furthermore, an experiment is conducted to test the measurement accuracy of the proposed method. At last, the assessment of workers' performance based on work analysis is discussed.

Chapter 5 summarizes the results of the study and proposes areas for further research. The expectation of worker assignment in the line production system using the proposed method is discussed.

Also, Figure 1.1 shows the organization of this thesis.

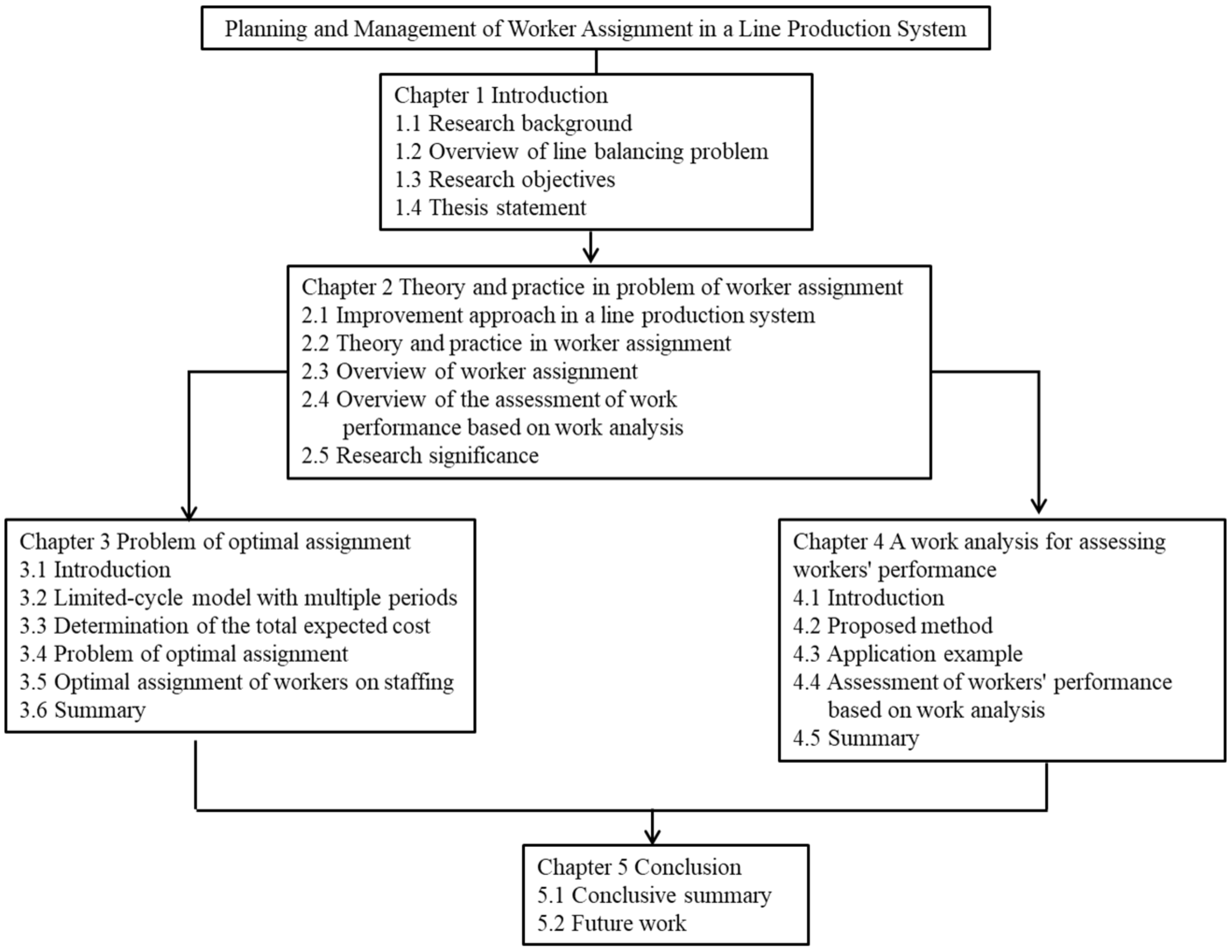


Figure 1.1: Organization of this thesis.

# Chapter 2 Theory and practice in problem of worker assignment

This chapter states the improvement approach in a line production system including the PDCA cycle, visualization, and the SECI model of knowledge dimensions. The procedure of theory and practice in worker assignment is then discussed. Additionally, prior relevant research, including research on worker assignment and assessment of work performance based on work analysis, is reviewed. Finally, the significance of the present study is given.

## 2.1 Improvement approach in a line production system

The problem of worker assignment is thought to lead to improved productivity ultimately. In managing a line production system, in addition to productivity, various indicators related to quality and delivery dates are checked in parallel from multiple perspectives, and members with knowledge in various fields are organized to improve each indicator. Approaches to improvement for multidimensional indicators are diverse, but typical approaches include the PDCA cycle, visualization, and the SECI model of knowledge dimensions.

(1) PDCA cycle

The PDCA cycle (plan-do-check-action cycle) proposed by Deming has been widely used in quality management[43]. The concept of the PDCA cycle refers to the continuous improvement of quality and service in a continuous improvement and cycle process, that is, under the guidance of a reasonable plan, the implementation of improvement activities using appropriate measures, the inspection and continuous tracking of results, the conclusion of the experience, the analysis of conclusions, and the next PDCA cycle improvement process. The PDCA cycle is an important way to identify, analyze, and solve problems, and is a logical guideline for doing so. It can be flexibly applied to many aspects of actual work.

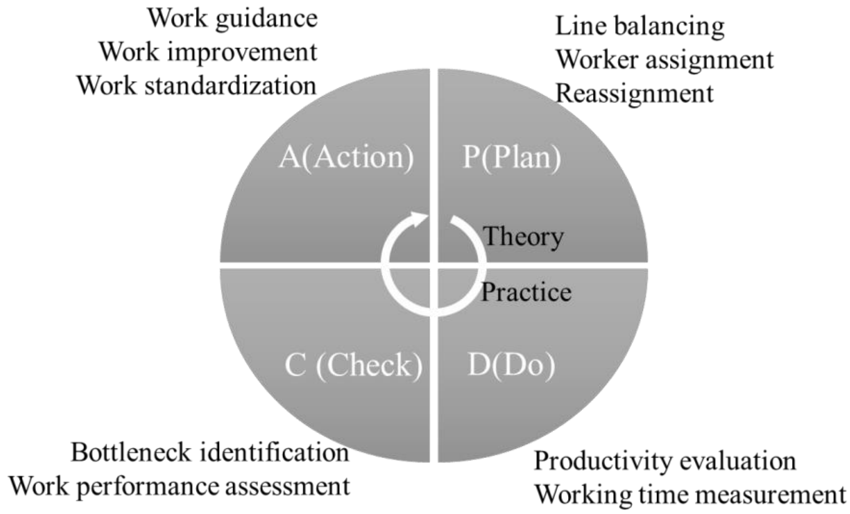


Figure 2.1: The approach to worker assignment problem from a PDCA cycle.

The PDCA cycle divides the activities necessary to consistently achieve a specific goal into four main phases and models the process of achieving the goal. When applied to a line production system, it is possible to continuously improve management indicators by repeating the PDCA cycle to improve quality including defect rate, cost including total expected cost, and delivery dates including work-in-process inventory. P(Plan) is to set up an improvement plan, D(Do) is to execute the plan, C(Check) is to measure performance and clarify the differences from the plan and the reasons for them, and A(Action) is considered as an activity to suppress the differences between the plan and the actual results[44].

Applying the PDCA cycle to the problem of worker assignment in a line production system, P (Plan) involves equalizing standard working time by line balancing and assigning or reassigning workers to each process in consideration of each worker's capability to perform the task at that time. D(Do) corresponds to the activities of executing production under the planned worker assignment, evaluating productivity, and measuring the actual working time for each process. C (Check) is an activity corresponding to assessing the actual work performance of each worker against the standard working time by comparing the actual working time of each process, as well as finding the bottleneck process[16].

Furthermore, A (Action) means providing work guidance and making work improvements with the intention that the work is executed according to the standard working time, and reviewing work standards and work procedures based on the work improvements. By continuing the PDCA cycle approach to assign workers as described above, total expected costs can be reduced, and other indicators can be improved. The above procedure is schematically expressed in Figure 2.1.

As shown in Figure 2.1, P(Plan) can be viewed as a theoretical model to find the worker assignment that minimizes the total expected cost by taking into account the quantified work capability of each worker under the work assignment equalized by line balancing. However, D(Do) involves measuring the working time of each process in reality. Thus, a method to measure the working time of each process is in need.

From the above point of view, it is required to have an optimal assignment in the theoretical model and to measure the working time in reality in order to practice the PDCA cycle. Furthermore, in improving quality management items (e.g., defect rate) and delivery date management items (e.g., work-in-process inventory), it is crucial to assign workers according to their work capabilities and to get a realistic view of the work being performed in each process.

(2) Visualization

Visualization is based on information sharing as an organizational activity rather than an individual activity. It is designed to allow all participants to see the plan, status, and results of the project, and to understand further challenges and directions for measures. Yamashita stated that visualization plays a role in motivating organizations to take action[45]. Following this approach, the theoretical model is used to plan worker assignments. Measuring the working time in reality through visualization provides insight into the capability of the workers performing the work in each process. Everyone in the organization is considering taking action to reduce the gap between planned and actual results by undertaking autonomous and proactive activities.

Therefore, in implementing worker placement, even from the viewpoint of visualization, which is practiced as a means of achieving goals at many production sites, turning worker assignment in the theoretical model and working time measurement in reality as a series of activities is considered to be an essential requirement.

(3) Knowledge creation

Knowledge creation is an activity in which diverse people with different specialties collaborate to create value[46]. The SECI model has been proposed as one approach to knowledge creation. The SECI model is theorized as a process of continuous knowledge creation through the mutual transformation of tacit and explicit knowledge. It models the process of upgrading the knowledge level of an organization by driving a chain of four modes of knowledge transformation: Socialization, Externalization, Combination, and Internalization (Figure 2.2)[47].

Explicit knowledge is expressed in words, numbers, etc. It is the knowledge that is objectively organized and easy to communicate and share. Tacit knowledge is knowledge that is personal, difficult to formalize, and difficult to communicate with others. The problem of worker assignment is considered to be knowledge creation, and the worker assignment approach is discussed by applying it to the SECI model.

Externalization is the process of verbalizing the tacit knowledge of individuals and sharing it with members of the organization. From the perspective of theoretical models, in addition to modeling the total expected cost and the worker assignment rule that minimizes the total expected cost, it corresponds to organizing the knowledge of the members and the data collected so far related to quality and delivery.

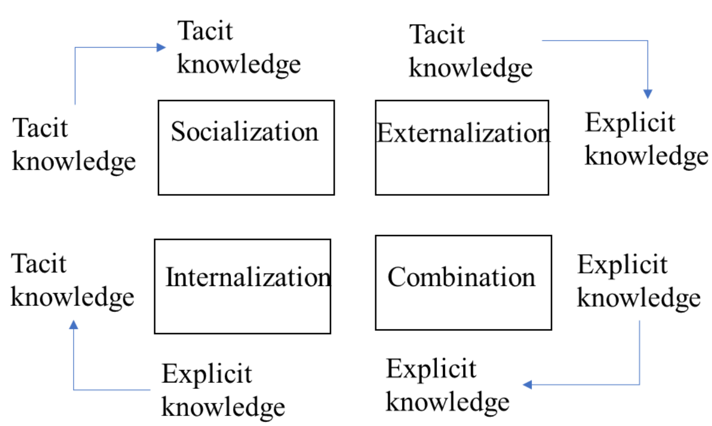


Figure 2.2: Knowledge creation approach by the SECI model.

Combination is the process of combining different knowledge to create new knowledge. It is an activity that examines the difference between the planned and actual results for each item and finds measures to improve QCD (quality, cost, and delivery)-related evaluation indicators, including worker reassignment, from the perspective of bottlenecks and minimizing total expected costs.

Internalization is the process of acquiring newly acquired explicit knowledge through learning. Workers assigned to each process carry out the prescribed work according to the procedure manual. It corresponds to proposing ideas for further working time and QCD through practice.

Socialization is the process of transferring tacit knowledge through common experiences. Activities such as the Quality Control Circle, etc., are held after working hours, and activities to share their creativity fall under this category. In the process of internalization to socialization, it is necessary to be able to explain the actions of each worker quantitatively to implement this process efficiently. Each person's actions provide information not only about working time but also about the information on management items related to quality and delivery. Therefore, from the viewpoint of knowledge creation, it is essential to perform work analysis in reality and obtain data for the implementation of the SECI approach.

In a synthesis of the above, various models have been proposed for approaches to improve production. Typical models include the PDCA cycle, which represents the process of reaching goals; visualization, which is intended to enable the understanding of challenges and directions for measures; and the SECI model of knowledge dimensions, which is one of the knowledge creation processes.

The ideas of these approach models also apply to the worker assignment problem. Based on these approach models, if the goal is to improve QCD-related management indicators including worker assignment problem, it is essential to achieve both: planning with theoretical models and work analysis that can measure the time difference between theory and practice.

## 2.2 Theory and practice in worker assignment

When assigning workers, it is considered that the working time changes sequentially due to the influence of proficiency. Therefore, while executing worker assignment, as mentioned in Section 2.1, workers are assigned to minimize the total expected cost in the abstract theoretical model. Worker assignment is based on work capability as a precondition. Then, work capability is measured in practice on a time-by-time basis, and when the preconditions change, the theoretical model is returned and workers are reassigned.

In a line production system, the procedure for assigning workers differs depending on whether the item to be produced is a new item or an existing product that has been produced in the past. In the case of an existing product, it can be assumed that the work capability of each worker is known in advance. Therefore, the worker assignment (optimal assignment) can be done first and the production line can start running. By measuring the working time, and predicting the working time for the next period, when the classification (category) of work capability changes, revert to the theoretical model and reassign workers. When the item to be produced is new, the working time is measured first using the product in the prototype stage, and the work capability of each worker is obtained by comparing it with the standard working time. After that, a theoretical model is used to plan worker assignments in consideration of work capability. As mentioned above, worker assignment is managed by a back-and-forth between theory and practice, i.e., planning in the theoretical model and measuring in the reality. The above two patterns of worker assignment procedures can be schematically illustrated in Figure 2.3.

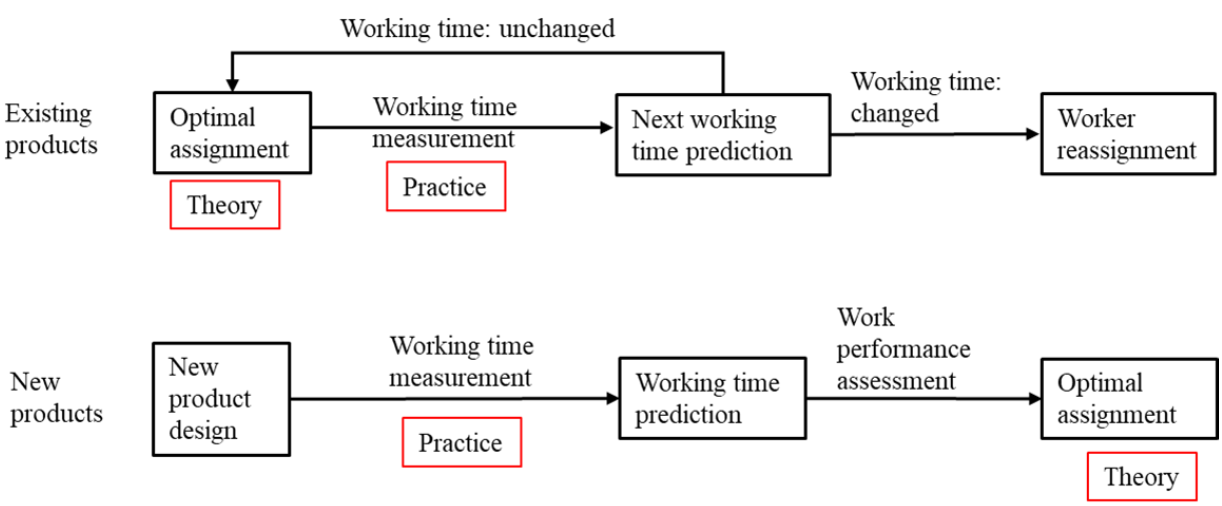


Figure 2.3: Image of the sequence of theory and practice of worker assignment.

## 2.3 Overview of worker assignment

### 2.3.1 The problem of worker assignment

Worker assignment means assigning employees to operations for the processing of parties[48]. The problem of optimal assignment is how to select the appropriate workers from the available workers to be responsible for each workstation. For the assignment problem where there is a task and exactly one worker to complete it, it is also called the equilibrium assignment problem, and for the assignment problem where the number of workers is more than the number of tasks, it is transformed into the equilibrium assignment problem by creating a fictitious task. Currently, the 0-1 planning method[22], the Hungarian method[49], and other methods can be used to solve such problems[50]. Van den Bergh et al. presented a review of the literature on worker assignment problems and identified trends and indicated areas[51]. However, considering the optimal assignment of workers on an assembly line often requires meeting the balance constraints of the assembly line[52] along with the assignment problem constraints[53], so its complexity is increased. For instance, Miralles et al. considered some task–worker incompatibilities in worker assignment[54]. Techawiboonwong et al. proposed classification by skilled and unskilled workers, permanent and temporary workers, and constructed an optimal worker assignment model for decision-making on the optimal type and number of workers to be used[55]. Song et al. considered the lowest standard deviation of working efficiency, the most efficient production line and the least waste in total working efficiency to find out the assignment of workers[56]. Suer and Tummaluri took learning and forgetting into account while assigned workers[57]. There is an increasing interest in solving worker assignments and assembly line balancing problems [58][59][60][61]. There is currently an obvious tendency toward including worker assignment in line balance.

This happens in an assembly line as follows: under uncertain situations, if a risk such as an idle or delay occurs in the production line, the result and efficiency of a certain production cycle period and a certain process are influenced, and the line balance will be lost. Some situations depend on the risks that have occurred previously. The minimum expected risk of the above situation where the risk depends on the previous situation and occurs repeatedly for multiple periods is focused on. Whether the process meets restriction usually depends on the state of the past process[62][63]. When the risk depends on the past processes, which assignment of tasks, workers or machines is most economical and efficient is an important issue in the risk plan[64][65].

In the limited-cycle model with multiple periods mentioned in Section 1.1, there are objects with certain constraints (cycle time) and these constraints will cause risks and the object occurs repeatedly for multiple periods. Since the capability to perform work is different for each worker, work may not be completed on time, idle time may occur, and a delay in a certain process may cause another delay in the following process. The problem that regards the loss of work time over multiple processes with this kind of cycle time constraint as a cost and discusses minimizing the total expected cost is called a limited-cycle problem with multiple periods[66].

The research related to the limited-cycle model with multiple periods was presented in Section 2.3.2, and these papers will be also used as a reference to further discuss the optimal assignment of workers in this model.

### 2.3.2 Related research on optimal assignment in the limited-cycle model with multiple periods

Yamamoto et al. considered how to solve such problems and generate the limited-cycle model with multiple periods[66]. This model is set with constraint, (e.g., target processing time) which is repeated in every multiple period. If the constraint is exceeded, the expected risk (e.g., penalty cost) will have occurred. And according to whether the constraint (target processing time) reset, the limited-cycle model with multiple periods is divided into a non-reset model and a reset model. In the limited-cycle problem with multiple periods, a model assuming that in the event of a work delay in a certain process, an adverse effect on the subsequent processes is prevented by increasing the number of relief workers, etc. is called a reset model. A model in which a work delay in each process has a direct impact on the subsequent processes is called a non-reset model.

The previous studies assumed that the processing capabilities of a worker on each process are the same. In the field of the non-reset model, Sun et al. considered the rules of optimal efficiency switching timing under the model of the non-reset limited-cycle model with multiple periods[67]. Sun et al. considered quality, production, and due date, to discover the optimal switching point to minimize the total expected cost of the parallel production system[68]. In the field of the reset model, Yamamoto et al. proposed a recursive formula for the expected risk and an optimal assignment algorithm based on the branch and bound method[69]. Besides, when the target processing time was constant, Yamamoto et al. [70] and Kong et al.[71] proposed some optimal assignment rules with two kinds of workers, among which one special worker exists. Also, Kong et al. proposed some rules of optimal assignment with two special slower or quicker workers[72]. Song et al. proposed some optimal assignment rules with three untrained workers[73] and Kong et al. considered the optimal assignment rules without concerning the number of minor untrained workers[74]. Furthermore, Kong et al. considered the optimization problem with three untrained workers where the consecutive delay time is limited[75]. Zhao et al. proposed some optimal assignment rules with three kinds of workers such as general workers, well-trained workers, and untrained workers[76]. Besides, Zhao et al. considered the situations where the target working time is variable[77].

These optimal assignment rules are all proposed on the premise that the workers have the same processing capabilities for different processes. These previous studies did not consider the difference in processing capabilities of the same worker for different processes. However, sometimes the processing capabilities of a worker on each process are different. Considering that there are two types of capabilities (poor or clever) in the process, Kong et al. proposed optimal assignment rules where n-1 workers have two clever processes, and 1 worker has no clever process[78].

In this thesis, the more general situations where the same worker has different processing capabilities for different processes are considered. Generally, each worker has his or her own strengths and weaknesses. The processing capabilities of a worker are variable when doing different processes. This thesis aims to focus on that worker has different capabilities for different processes under the reset limited-cycle model with multiple periods.

## 2.4 Overview of the assessment of work performance based on work analysis

### 2.4.1 The assessment of work performance

The current understanding of performance is mainly considered from two perspectives: results and behavior. Bernardin et al. suggested that performance should reflect the set of results of all tasks performed by a worker in the task to be assessed[79]. Kane et al. argued that performance has some independence and is a legacy product of workers completing a task while following a certain guideline[80]. Understanding performance in terms of results facilitates the clarity of the purpose of performance assessment and the establishment of the assessment index system, making performance assessment clear. However, to a certain extent, it can lead to biased assessment results due to an insufficient understanding of the workers' work process, which in turn ignores important factors in the performance assessment process. Another view is that performance is a set of behaviors that workers perform to get their tasks done[81]. Campbell similarly described the performance as actions or behaviors that are pertinent to the organization's objectives and that may be assessed in terms of the degree to which they contribute to those objectives[82]. It is possible to distinguish between these behaviors and effectiveness, which is the influence these actions have on results. He also views technical skill performance as the foundation of a person's job-specific task competency. This type of view broadens the perspective of evaluation, shifting assessment management from a simple focus on results only to a comprehensive analysis and consideration of the worker's work process. Next, after considering both work behavior and work results together, performance is understood with the view that the integration of behavior and results in work performance[83][84].

It is worth discussing how to manage the performance of workers in the production process so that both the individual level of workers and the overall level of the production system can be improved. McAFee and Champagen stated that performance management is composed of a cycle that starts with performance and goes through planned performance - management performance - assessment performance and back to planned performance[85]. Planned performance is to be based on the objectives of the production system. By using various scientific qualitative or quantitative methods to assess and evaluate the behavior of workers in the process and results, that is, to assess the work performance of workers, and to make objective and fair adjustments in on-site management[86]. Therefore, workers and managers need to know the work objectives, and they need to assess the performance of workers against the objectives to know whether the work is being performed properly according to the plan. It is through the monitoring of work results that performance can be assessed, and optimal adjustments can be made to worker behavior[87]. In practical terms, how to assess performance is a challenge. In this thesis, the assessment of workers' performance using a work analysis is considered.

### 2.4.2 Related research of work analysis

Work analysis is a series of methods to identify the content of a work based on the activities involved in the work and the characteristics or requirements necessary to conduct those activities[88]. The work analysis is used to determine the motion and the time required for a worker to complete a task so that an assessment can be made as to whether the worker is performing the work according to the specified standards.

Traditionally, video analysis has been used to analyze work time for various tasks. While video analysis is used extensively in work analysis[89], a substantial amount of time is typically needed for the analysis to be conducted manually. Video analysis generally requires a substantial amount of time and labor to analyze the videos and identify each operation in detail, which makes it too inefficient for real-time work improvement. To address this, automation of work analysis has been researched. Previous studies related to automatic work analysis can be divided into two categories: those focused on the measurement of a movement line and those focused on motion analysis.

Research focusing on indoor positioning and tracking has been widely reported. In one previous study, RFID (Radio Frequency Identification) was used for positioning; however, to apply this method in a large indoor facility comes at a significant cost, since multiple RFID antennas are required[90]. Image processing methods are also common in many tracking systems. Compared to the RFID method, the cost of an image processing device is lower, but measurement accuracy can be seriously affected by changes in the lightening environment at different times of the day or as the result of changing weather conditions[91]. Ultrasonic sensors have also been used in indoor dynamic positioning systems, yielding better measurement accuracy than an RFID system at a lower total cost of materials and equipment[92]. However, limitations on the configuration of the measurement area give rise to an important issue when using ultrasonic sensors, as the transmission of ultrasonic signals can be easily influenced by obstacles such as pillars and walls. Another dynamic positioning method has been proposed for an over-the-horizon communication environment using a tracking camera. With this method, one or two cameras are attached to the tracking camera and a trace of the target is obtained through VSLAM (Visual Simultaneous Localization and Mapping)[93]. Two different snapshots are taken either by one moving camera or by two different cameras to establish multiple image feature points; the corresponding feature values are determined by applying the SIFT method (Scale-Invariant Feature Transform)[94] or FAST method (Features from Accelerated Segment Test)[95]. Feature points with the same feature values are then identified in the two snapshots, and the three-dimensional coordinates of these feature points are determined using the DLT method (Direct Linear Transformation)[96]. The moving distance during one frame can be calculated with the template-matching method using the different feature values of two adjacent frames with respect to these feature points. The moving distance of the tracking camera is then calculated using an inverse calculation of the moving distance of the identified feature points in the three-dimensional space. With this procedure, the position of the tracking camera in the three-dimensional space will be updated with each frame. Although this tracking camera-based method is suitable for positioning in an environment with multiple obstacles such as pillars or shelves, it cannot be used for long-term measurement since the measurement error accumulates with time.

In studies involving motion analysis, a motion capture device is often used to measure different motions[97]. However, these devices typically cover the entire body, which makes them difficult to wear for an extended time during work. Recently, Microsoft's Kinect has been proposed for motion analysis[98]; however, its measurement range is limited since, with this method, a depth image is used for detection. When the measuring range is broad or the environment is in direct sunshine, it is not applicable.

Ideally, what is needed for efficient work analysis in assembly line is a system suitable for a broad work area with many narrow passages, where ultrasonic signals are easily sheltered, using a motion capture device that is easy to wear and is less of a hindrance to the worker carrying out his/her routine.

Accordingly, this thesis proposes an automatic work analysis method that works with various devices, including an ultrasonic sensor network, a tracking camera and an acceleration sensor, able to cover a wide work area for accessing workers' performance in assembly line.

## 2.5 Research significance

Theory and practice are combined to plan and manage the worker assignment in a line production system. From the viewpoint of productivity improvement in the management of the line production system, worker assignment theory and practice are continually evolving. The planning and reassignment of worker assignment in theory, work analysis in practice, the assessment of the discrepancy between the planned and actual work capability, and the improvement will be continued. The worker assignment approach will become more practical as a result.

In the research of worker assignment, mathematical models are usually used to consider the planning of worker assignment. Taking into account the actual situation, various factors are taken into account in the model to plan worker assignment, and the algorithm is applied to optimize worker assignment. Building a mathematical model that can simulate the conditions of an actual production line and simplifying an actual production line in a specific way are frequent study methods. Due to the different factors considered, the objective function established varies. For the planning of worker assignment in this thesis, a model called the limited-cycle model with multiple periods is mainly considered. The objective of this model is to reduce total cost while taking into account delay cost and idle cost. After that, the mathematical model is solved appropriately to produce the theoretically ideal assignment for the line designer's reference. In this thesis, it is considered that even if the worker assignment is optimized by production line balancing, the working time of workers may change due to the factors of proficiency and fatigue, making the originally balanced production line unbalanced. Therefore, when optimizing the worker assignment, the variability of the processing capability of workers is considered, the rule of optimal assignment that can be used in the design of production systems is proposed, and the method of applying the rule of optimal assignment to the staffing is discussed.

In the practice of worker assignment, turning worker assignment in the theoretical model and working time measurement in reality as a series of activities is considered to be an essential requirement. The work analysis system is designed to make it easier to switch between worker assignment in the theoretical model and in practice. Since video analysis is too time-consuming and inefficient, an automated work analysis system is proposed. The work analysis system measures the working time necessary for assessing work performance. The system can detect the effect of the implementation of work through time measurement and motion analysis. It can help managers to quickly grasp the accurate daily production situation; when the actual capability does not meet the requirements of the planned capability, it can help managers to find out where the problems lie so that they can better make management decisions based on scientific arrangements for production planning and other work. The work analysis automation method is aimed at measuring the actual working time of each process and contributing to the re-evaluation of the work capability of each worker and the redesign of worker assignments.

Combining the actual processing capability of workers with the production plan of the assembly line can provide the basis for optimizing the staffing on the assembly line by understanding and grasping the completion of workers' tasks.

# Chapter 3 Problem of optimal assignment

## 3.1 Introduction

In line balancing, the cycle time is generally determined first, and then tasks are assigned to minimize the number of processes by using that number as a variable. Another option is to decide on the number of processes first, then divide up the tasks to reduce cycle time. In addition, the number of processes and cycle time may also be variables at the same time, and work assignments may be performed to keep both to a minimum. This chapter assumes that the number of processes is minimized for a known cycle time before planning the worker assignment, and no modification of the cycle time occurs during the planning of the worker assignment.

Chapter 3 focuses on the problem of optimal assignment in a reset limited-cycle model with multiple periods. In the line balancing problem, the load of each process is equalized with a known target time and antecedent relationship for each task. Next, workers are assigned to do the proper tasks. In practical terms, the condition of differences in worker proficiency can lead to different workers who may perform the same task with different results. In addition, every worker has their strong points and weak points according to the tasks assigned. Therefore, it is suited to the actual circumstances if each worker's level is set for each process. Then the problem of optimal assignment is modeled to minimize the idle cost of finishing earlier than the target time and the penalty cost of finishing later. In production activities, the result and efficiency of a certain period of a production cycle are influenced not only by the risks that exist in the current period but also by the risks that existed in the previous ones. A model, called a reset limited-cycle model with multiple periods, to assume evaluating the impact risk is constructed. Because of the variability of the capabilities of workers, the working time of the same task is different among workers. This chapter concentrates on the fact that workers have different processing capabilities for different processes in the reset limited-cycle model with multiple periods. Based on that assumption, a new formulation of the expected risk(cost) is proposed. Furthermore, some theorems of the rule of the optimal worker assignment that minimizes the total risk are proposed when there are two types of workers' capabilities for each process. Finally, a method for staffing based on the optimal assignment rules is discussed.

## 3.2 Limited-cycle model with multiple periods

In this section, first, a reset limited-cycle model with multiple periods is explained. Then, some assumptions are described when different workers' capabilities for each process in the reset limited-cycle model with multiple periods. Finally, the problem of optimal assignment in the reset limited-cycle model with multiple periods is presented.

### 3.2.1 Limited-cycle model with multiple periods

This section considers an assembly line in which the initial input materials are processed sequentially in a series of processes to produce a finished product at the end. Delays in one period will affect subsequent periods, leading to late deliveries and increased costs. Similarly, the risk of delay in a given period is not only dependent on that period, but also on previous periods. A model that assumes that risk depends on past conditions and that risk occurs repeatedly over multiple periods is called a limited-cycle model with multiple periods. As shown in Figure 3.1, materials are put into the assembly line and processed sequentially, starting with process 1 and ending with process $n$. In each process, the established cycle time (target working time) is a constraint condition. This constraint (target working time) is used to determine whether idle or delay occurs. The limited-cycle model with multiple periods can be divided into the following two models, depending on whether the start time of work in the subsequent process is affected.

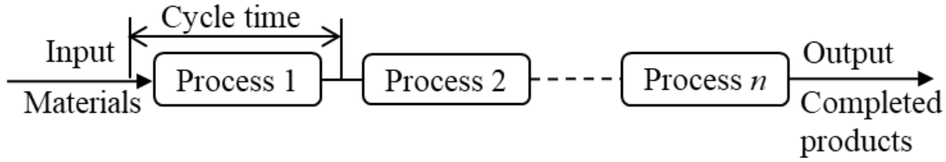


Figure 3.1: The production line in the limited-cycle model with multiple periods.

In a non-reset limited-cycle model with multiple periods, if the processing is delayed in one step, the next process is immediately impacted. In other words, this is a model in which the beginning of each process occurs simultaneously with the end of the one before it.

In a reset limited-cycle model with multiple periods, if the processing is delayed in one step, the next process is not affected by the status of the previous process by increasing the number of relief workers, etc. In other words, the work time is reset on the model. In this chapter, the reset limited-cycle model with multiple periods (hereinafter referred to as the reset model) is discussed.

### 3.2.2 Assumptions and symbols

The model is considered based on the following assumptions with different workers' capabilities for each process.

In an assembly line system, *n* is the number of processes. The production is processed in a rotation of process 1, process 2, ..., and process *n*. One production will be processed by all *n* processes.

All the partly finished products will be moved to the next period and processed within time *Z*. Especially *Z* is the cycle time of all periods. *Z* is also a kind of limited working time for each period. *Z* is called target working time.

Due to the various processing capabilities of workers, the actual working time cannot always follow the limit of target working time *Z*. This model also needs to consider idle and delay. The working time of workers is self-dependent. The processing capability is decided by the property of the worker own and is not influenced by the processing status such as idle or delay.

There are *n* workers. One worker is assigned to each process. Only one worker can be assigned to each process. And each process must be assigned by one worker. The following assumptions are the original content of this thesis. The process number is $i$ and the worker number is $j$.

For process $i$, the working time of worker $j$ is $T_{ij}$.

$P_{ij}$ : The probability of worker $j$ becoming idle, which is $\Pr\{T_{ij} \le Z\}$,

$Q_{ij}$ : The probability of worker $j$ becoming delayed, which is $\Pr\{T_{ij} > Z\}$,

$TS_{ij}$ : The expected idle time of the worker $j$ , which is $E[(Z - T_{ij})I(T_{ij} \le Z)]$,

$TL_{ij}$ : The expected delay time of the worker $j$ ,which is $E[(T_{ij} - Z)I(T_{ij} > Z)]$ ,

where $I\ (\bullet)$ is an index function and given as follows:

$$I(O)=\begin{cases}1 & (O \text{ is true}) \\ 0 & (O \text{ is not true}).\end{cases}$$

It is assumed that working time follows either probability distribution, such as exponential distribution or normal distribution. For process $i$, the working time probability density function of worker $j$ is $f_{ij}(t)$.

$$P_{ij} = \int_0^Z f_{ij}(t)dt$$
$$Q_{ij} = \int_Z^\infty f_{ij}(t)dt$$
$$TS_{ij} = \int_0^Z (Z-t) f_{ij}(t)dt$$
$$TL_{ij} = \int_Z^\infty (t-Z) f_{ij}(t)dt\ .$$

$P_{ij}$, $Q_{ij}$, $TS_{ij}$ and $TL_{ij}$ are used to distinguish between workers' capability.

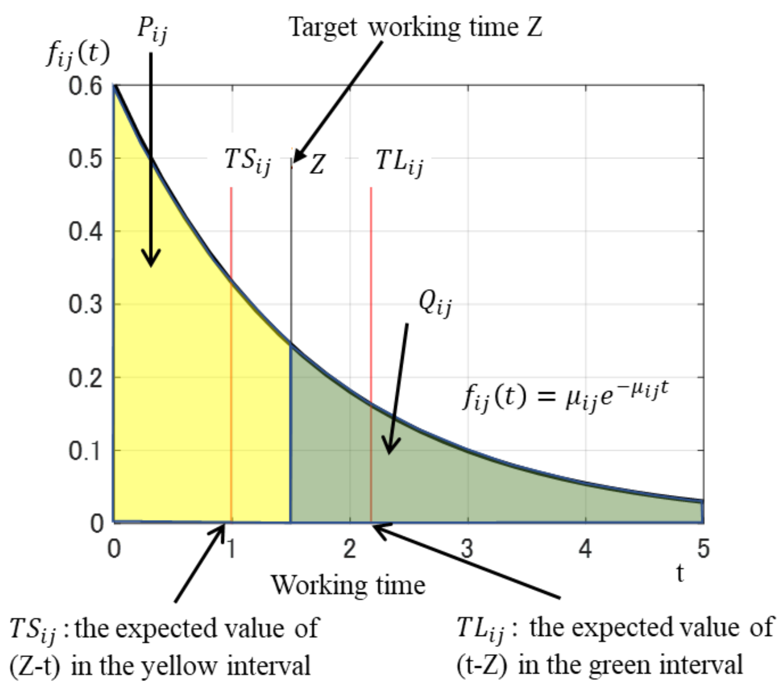


Figure 3.2: Example of $P_{ij}$, $Q_{ij}$, $TS_{ij}$ and $TL_{ij}$

As an example, Figure 3.2 shows a schematic diagram of idle probability and delay probability when the working time is assumed to follow an exponential distribution. The probability that the working time is within *Z* is obtained from the cumulative probability of the interval 0<t<*Z*, and is used as the idle probability. Furthermore, in this interval, the expected value of the time difference from the working time to *Z* is obtained as the expected idle time. Based on the same concept, set the delay probability and the expected delay time when the working time is longer than *Z*.

In this model, a regular processing cost $C_t(>0)$ per unit time will permanently occur during target working time *Z*, regardless of whether it is delay or idle. It is for the reason that although the production is accomplished prematurely in current process, the next process may be occupied by another production. Therefore, the production must wait for its start. And as a result, the idle cost will occur. On the other hand, if the working time is longer than *Z*, it is supposed that the delay of process time can be recovered by the overtime work or spare workers in this process, and as a result overtime work or additional resources will be requested to meet the target time *Z*. So the delay cost will occur.

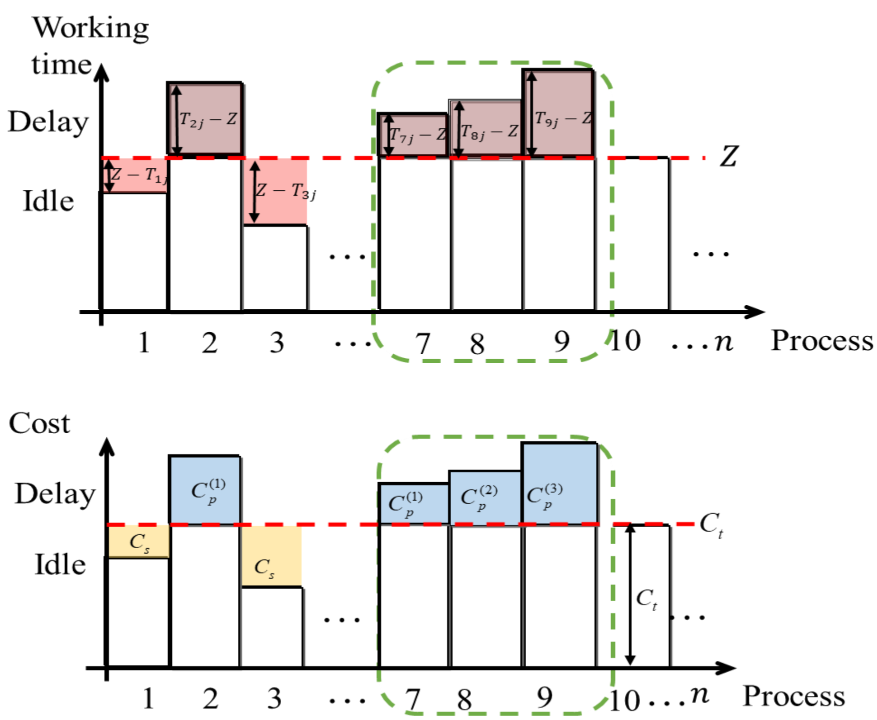


Figure 3.3: Working time and cost in reset model.

The processing cost per unit time, $C_t (> 0)$, for the target working time limit occurs in each process. When $T_{ij} \leq Z$, the idle cost per unit time $C_s (\geq 0)$ occurs in each process. When $T_{ij} > Z$, the delay cost per unit time $C_P^{(\alpha)} (> 0)$ occurs in a period if delay occurs in $\alpha$ consecutive processes before the process, for $1 \leq \alpha \leq n$. In this research, it assumes that the process delay time during the period can be recovered by overtime work or additional workers in the period, and $C_P^{(\alpha)}$ is the cost for all of these. If the delay continues for several processes, it can be considered that it will cost more to recover the delay. When continuous delays occur, the continuous delay cost per unit time is set higher than in the case of a single process delay because of the need to increase the number of additional workers. Also, in a production line with a short cycle time of several tens of seconds, when a continuous delay occurs, all processes may be stopped until the continuous delay is resolved. Because the cost rises due to the increase in the delay, it supposes in this research that $C_P^{(\alpha)}$ is monotonically increasing on α; that is $0 \leq C_P^{(1)} ... \leq C_P^{(n)}$.

Figure 3.3 is used as an example for illustration. It represents the working time of seven processes that all have a target working time *Z*. All these processes incur a cost of $C_t Z$. The working time of process 1 is shorter than *Z*, so additional idle costs are incurred. The working time of process 2 is longer than *Z*, so additional delay costs are incurred. Similarly, process 3 is the same as process 1, and process 7 is the same as process 2. Process 8 takes longer to work on than *Z*, so additional costs are incurred. But since process 7 is also delayed, consecutive delays are likely to make the number of relief worker increase, so the cost of delay per unit time on the eighth project may increase compared to process 7. This is especially true for process 9 as the third process in a consecutive delay. No additional costs are incurred in process 10 since the task is finished on time.

### 3.2.3 Problem of optimal assignment in a reset model

The problem of optimal assignment in a reset model is how to assign the workers to minimize the total expected cost. The assumptions and symbols used in the problem of optimal assignment are defined as follows.

For $1 \leq i \leq n$,

$\pi(i)$ : The number of worker working on process $i$.

$\pi$ : A $n$ dimensional vector, where elements are $\pi(i)$, that means the assignment of workers.

For example, when assigning worker $\pi(1)$ to process 1, worker $\pi(2)$ to process 2,…,worker $\pi(n)$ to process $n$, then the assignment of the system can be represented as $\pi = (\pi(1), \pi(2), \pi(3) \cdots \pi(n))$

$TC(n;\pi)$ : The total expected cost from process 1 to process $n$ when workers are assigned in assignment $\pi$ .which can be expressed as

$$TC(n;\pi) = nC_t Z + f(n;\pi) \tag{3.2.1}$$

here,

$f(n;\pi)$ : The expected cost (the sum of the expected delay cost and the expected idle cost caused in process $n$).

By using these symbols, the problem of optimal assignment becomes the problem of getting an assignment in the following equation:

$$TC(n;\pi^*) = \min_{\pi} TC(n;\pi) \tag{3.2.2}$$

However, it is known from (2.2.1) that the target production cost $nC_t Z$ is constant if the target working time $Z$ is constant. Eq. (3.2.2) can be simplified to

$$f(n;\pi^*) = \min_{\pi} f(n;\pi) \tag{3.2.3}$$

where $\pi^*$ is called the optimal assignment.

The value obtained by multiplying the probability of delay by the delay cost per unit time and the expected delay time is included in the total expected cost as the expected delay cost. The value obtained by multiplying the probability of idle by the idle cost per unit time and the expected idle time is included in the total expected cost as the expected idle cost. In this thesis, the bottleneck process is considered to correspond to a process with a large expected delay time. It can be said that the concept of minimizing the total expected cost encompasses the idea of designing highly productive worker assignments by focusing on the bottleneck process.

## 3.3 Determination of the total expected cost

In this section, a method is proposed for calculating the total expected cost where the workers have different processing capabilities for different processes. First, the total expected cost formula where each worker has a different processing capability for all processes is summarized as a theorem. After that, a formulation of the total expected cost is proposed where there are two types of processing capabilities of a worker for each process.

### 3.3.1 In the case where each worker has a different processing capability for all processes

A method is proposed for calculating the total expected cost where each worker has a different processing capability for all processes. The expected cost can be obtained by counting all states of whether each process is idle or delayed and calculating the probability and expected time of that state. Here, a formulation of the total expected cost is proposed using a stochastic model.

**Theorem 1**

For $i = 1, 2 \cdots n$, the sum of the total expected cost $TC(n;\pi)$ in Eq. (3.2.1) can be expressed as

$$\begin{aligned} &TC(n;\pi) \\ &= nC_t Z + f(n;\pi) \\ &= nC_t Z + \sum_{i=1}^{n} C_s TS_{i,\pi(i)} + \sum_{i=1}^{n}\sum_{\alpha=1}^{i} C_p^{(\alpha)} TL_{i,\pi(i)} D(i,\pi(i);\alpha) \end{aligned} \tag{3.3.1}$$

where,

$$D(i,\pi(i);\alpha) \equiv \begin{cases} 1 & i = 1 \\ P_{i-\alpha,\pi(i-\alpha)} & i > 1, \alpha = 1 \\ P_{i-\alpha,\pi(i-\alpha)} \times \prod_{\beta=i-\alpha+1}^{i-1} Q_{\beta,\pi(\beta)} & i > 1, 1 < \alpha < i \\ \prod_{\beta=1}^{i-1} Q_{\beta,\pi(\beta)} & i > 1, \alpha = i \end{cases} . \tag{3.3.2}$$

Here, α is the number of consecutive delays in the process.

Eq. (3.3.1) is an extended version of the formulation proposed by Yamamoto et al[66].

**Proof of Theorem 1**

In Eq. (3.2.1), the target production cost $nC_t Z$ is constant, so the main concerns in the proof are on how $f(n;\pi)$ is calculated. $f(n;\pi)$ can be obtained by calculating the cumulative cost in the assignment $\pi$ in the state where it exists in all the processes from the 1st process to the *n*th process. All states are listed for each process, where ○

indicates idle and ● indicates delay.

For the $i$ th process, cases are classified into states $1 \sim i+1$, and the corresponding expected cost due to idle and delay are shown in Table 3.1.

When there are $n$ processes, the cost in the state from the first process to the $n$th process is accumulated. As a sum of these, the expected cost due to idle and delay that occurred is expressed as

$$
\begin{aligned}
& f(n;\pi) \\
&= f(n;\pi(1),\pi(2),\pi(3),\cdots\pi(i),\cdots,\pi(n)) \\
&= C_s TS_{1,\pi(1)} + C_p^{(1)} TL_{1,\pi(1)} \\
&\quad + C_s TS_{2,\pi(2)} + C_p^{(1)} TL_{2,\pi(2)} P_{1,\pi(1)} + C_p^{(2)} TL_{2,\pi(2)} Q_{1,\pi(1)} \\
&\quad + C_s TS_{3,\pi(3)} + C_p^{(1)} TL_{3,\pi(3)} P_{2,\pi(2)} + C_p^{(2)} TL_{3,\pi(3)} P_{1,\pi(1)} Q_{2,\pi(2)} + C_p^{(3)} TL_{3,\pi(3)} Q_{1,\pi(1)} Q_{2,\pi(2)} \\
&\quad + \cdots \\
&\quad + \left( \begin{array}{l}
C_s TS_{n,\pi(n)} + C_p^{(1)} TL_{n,\pi(n)} P_{n-1,\pi(n-1)} + C_p^{(2)} TL_{n,\pi(n)} P_{n-2,\pi(n-2)} Q_{n-1,\pi(n-1)} \\
+ C_p^{(3)} TL_{n,\pi(n)} P_{n-3,\pi(n-3)} Q_{n-2,\pi(n-2)} Q_{n-1,\pi(n-1)} \\
+ \cdots \\
+ C_p^{(n-2)} TL_{n,\pi(n)} P_{2,\pi(2)} Q_{3,\pi(3)} Q_{4,\pi(4)} \cdots Q_{n-1,\pi(n-1)} \\
+ C_p^{(n-1)} TL_{n,\pi(n)} P_{1,\pi(1)} Q_{2,\pi(2)} Q_{3,\pi(3)} \cdots Q_{n-1,\pi(n-1)} \\
+ C_p^{(n)} TL_{n,\pi(n)} Q_{1,\pi(1)} Q_{2,\pi(2)} Q_{3,\pi(3)} \cdots Q_{n-1,\pi(n-1)}
\end{array} \right) \\
&= \sum_{i=1}^{n} C_s TS_{i,\pi(i)} + \sum_{i=1}^{n} \sum_{\alpha=1}^{i} C_p^{(\alpha)} TL_{i,\pi(i)} D(i,\pi(i);\alpha)
\end{aligned}
\tag{3.3.3}
$$

where α is the number of consecutive delays in the process.

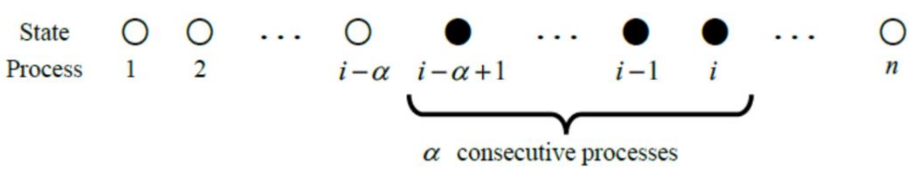


Figure 3.4: Image of α consecutive processes.

Table 3.1: State and the expected cost for the $i$ th process.

| *Process* | *1* | *2* | *3* | … | *i – 3* | *i – 2* | *i – 1* | *i* | *Cost for the* $i^{th}$ *process* |
|---|---|---|---|---|---|---|---|---|---|
| State 1 | | | | | | | | ○ | $C_s TS_{i,\pi(i)}$ |
| State 2 | | | | | | | ○ | ● | $C_p^{(1)} TL_{i,\pi(i)} P_{i-1,\pi(i-1)}$ |
| State 3 | | | | | | ○ | ● | ● | $C_p^{(2)} TL_{i,\pi(i)} P_{i-2,\pi(i-2)} Q_{i-1,\pi(i-1)}$ |
| State 4 | | | | | ○ | ● | ● | ● | $C_p^{(3)} TL_{i,\pi(i)} P_{i-3,\pi(i-3)} Q_{i-2,\pi(i-2)} Q_{i-1,\pi(i-1)}$ |
| … | | | | | | | | | |
| State $i-1$ | | ○ | ● | … | ● | ● | ● | ● | $C_p^{(i-2)} TL_{i,\pi(i)} P_{2,\pi(2)} Q_{3,\pi(3)} Q_{4,\pi(4)} \cdots Q_{i-1,\pi(i-1)}$ |
| State $i$ | ○ | ● | ● | … | ● | ● | ● | ● | $C_p^{(i-1)} TL_{i,\pi(i)} P_{1,\pi(1)} Q_{2,\pi(2)} Q_{3,\pi(3)} \cdots Q_{i-1,\pi(i-1)}$ |
| State $i+1$ | ● | ● | ● | … | ● | ● | ● | ● | $C_p^{(i)} TL_{i,\pi(i)} Q_{1,\pi(1)} Q_{2,\pi(2)} Q_{3,\pi(3)} \cdots Q_{i-1,\pi(i-1)}$ |

$$D(i,\pi(i);\alpha) \equiv \begin{cases} 1 & i=1 \\ P_{i-\alpha,\pi(i-\alpha)} & i>1, \alpha=1 \\ P_{i-\alpha,\pi(i-\alpha)} \times \prod_{\beta=i-\alpha+1}^{i-1} Q_{\beta,\pi(\beta)} & i>1, 1<\alpha<i \\ \prod_{\beta=1}^{i-1} Q_{\beta,\pi(\beta)} & i>1, \alpha=i \end{cases}$$

$f(n;\pi)$ and $nC_t Z$ are the two parts of Eq. (3.3.1). Therefore, Theorem 1 was proved.

$D(i,\pi(i);\alpha)$ is given by Eq. (3.3.2).

For $i=1$, $D(i,\pi(i);\alpha)=1$.

For $i>1$, when workers are assigned in the $\pi$, $D(i,\pi(i);\alpha)$ is the probability that idle occurs in the process $i-\alpha$, and delay occurs in all the processes from the process $i-\alpha+1$ to the process $i$ .This is illustrated in Figure 3.4, where ○ indicates idle and ● indicates delay.

### 3.3.2 In the case where each worker has two types of processing capabilities for each process

The total expected cost is calculated on the assumption that there are two types of processing capabilities (proficient and inefficient) of each worker for each process in the reset model. A worker with proficient capabilities can complete work in the processes according to a standard schedule while a worker with inefficient capabilities needs more time than the standard scheduled time to complete the work of the processes. Also, the following symbols are defined.

- $y$ : The number of workers who work on their proficient tasks in assignment $\pi$.
- α : The number of consecutive delays in the process.
- $A(i,\alpha)$ : The number of process that the worker is proficient at process $i-\alpha+1$ to process $i-1$ in assignment $\pi$.
- $B(i,\alpha)$ : The number of process that the worker is inefficient at process $i-\alpha+1$ to process $i-1$ in assignment $\pi$.

Two representative probability density functions are predefined for the working time in the inefficient and proficient cases.

For $i=1,2\cdots n$ and $j=1,2\cdots n$, $g_{ij}$ is an index function and given as follows:

$$g_{ij}=\begin{cases}0 & \text{When worker } j \text{ is inefficient at process } i\\ 1 & \text{When worker } j \text{ is proficient at process } i\end{cases}$$

For each process, the probability of idle and the probability of delay are compared, and if $Q_{ij}>P_{ij}$, the probability of a delay occurring is high, so $g_{ij}=0$ is determined to be inefficient. Otherwise, it is judged as $g_{ij}=1$. After that, the level of work capability of each worker is replaced with a probability density function that represents a predetermined level.

The probability of worker $j$ becoming delayed $Q_{ij}$ is defined as

$$Q_{ij}\equiv\begin{cases}Q_1 & g_{ij}=0\\ Q_2 & g_{ij}=1\end{cases}\qquad (Q_1>Q_2).$$

That is, if $g_{ij}=0$, then $Q_{ij}=Q_1$, it means that worker $j$ is inefficient at process $i$. If $g_{ij}=1$, then $Q_{ij}=Q_2$, it means that worker $j$ is proficient at process $i$.

The probability of worker $j$ becoming idle $P_{ij}$ is defined as

$$P_{ij}\equiv\begin{cases}P_1 & g_{ij}=0\\ P_2 & g_{ij}=1\end{cases}\qquad (P_2>P_1).$$

That is, if $g_{ij}=0$, then $P_{ij}=P_1$, it means that worker $j$ is inefficient at process $i$. If $g_{ij}=1$, then $P_{ij}=P_2$, it means that worker $j$ is proficient at process $i$.

Similarly, the expected delay time is defined as

$$TL_{ij} \equiv \begin{cases} TL_1 & g_{ij} = 0 \\ TL_2 & g_{ij} = 1 \end{cases} \qquad (TL_1 > TL_2)\,.$$

The expected idle time is defined as

$$TS_{ij} \equiv \begin{cases} TS_1 & g_{ij} = 0 \\ TS_2 & g_{ij} = 1 \end{cases} \qquad (TS_2 > TS_1)\,.$$

In other words, if worker $j$ is inefficient at process $i$,

$$Q_{ij} \equiv Q_1\,,$$

$$P_{ij} \equiv P_1\,,$$

$$TL_{ij} \equiv TL_1\,,$$

$$TS_{ij} \equiv TS_1\,.$$

If worker $j$ is proficient at process $i$,

$$Q_{ij} \equiv Q_2\,,$$

$$P_{ij} \equiv P_2\,,$$

$$TL_{ij} \equiv TL_2\,,$$

$$TS_{ij} \equiv TS_2\,.$$

In this case,

$$Q_1 > Q_2\,,$$

$$P_2 > P_1\,,$$

$$TL_1 > TL_2\,,$$

$$TS_2 > TS_1\,.$$

Under the above conditions, Theorem 2 is established.

**Theorem 2**

For $i = 1,2\cdots n$, the total expected cost $TC(n;\pi)$ can be expressed as

$$
\begin{aligned}
&TC(n;\pi)\\
&= nC_tZ + f(n;\pi)\\
&= nC_tZ + C_s((n-y)TS_1 + yTS_2)\\
&\quad + \sum_{i=1}^{n}\begin{pmatrix} TL_1(1-g_{i,\pi(i)})(C_p^{(1)} + \sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \\ + TL_2 g_{i,\pi(i)}(C_p^{(1)} + \sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \end{pmatrix}
\end{aligned}
\tag{3.3.4}
$$

**Proof of Theorem 2**

When $g_{ij} = 0$ (worker $j$ is inefficient at process *i*), the probability of worker *j* becoming delayed $Q_{ij} = Q_1$ . When $g_{ij} = 1$ (worker $j$ is proficient at process *i*), $Q_{ij} = Q_2$ . Expressing the probability of delay as one expression is

$$Q_{ij} = (1 - g_{ij})Q_1 + g_{ij}Q_2$$

Similarly, the probability of idle, the expected delay time and the expected idle time are

$$
\begin{aligned}
P_{ij} &= (1 - g_{ij})P_1 + g_{ij}P_2\\
TL_{ij} &= (1 - g_{ij})TL_1 + g_{ij}TL_2\\
TS_{ij} &= (1 - g_{ij})TS_1 + g_{ij}TS_2
\end{aligned}
$$

The target production cost $nC_tZ$ is constant, so the main concerns in the proof are on how $f(n;\pi)$ is calculated. $f(n;\pi)$ can be obtained by calculating the cumulative cost in the assignment $\pi$ in the state where it exists in all the processes from the 1st process to the $n$th process. All states are listed for each process, where ○ indicates idle and ● indicates delay.

For the $i$ th process, cases are classified into states $1 \sim i+1$, and the corresponding expected cost due to idle and delay are shown in Table 3.2.

Table 3.2: State and expected cost for the $i$ th process (the case where there are two types of processing capabilities of each worker).

| *Process* | *1* | *2* | *3* | … | *i* – *3* | *i* – *2* | *i* – *1* | *i* |
|---|---|---|---|---|---|---|---|---|
| State 1 | | | | | | | | ○ |
| | Cost for the *i*th process $C_s((1-g_{i,\pi(i)})\,TS_1+g_{i,\pi(i)}TS_2)$ | | | | | | | |
| State 2 | | | | | | | ○ | ● |
| | Cost for the *i*th process $C_p^{(1)}\left((1-g_{i,\pi(i)})TL_1+g_{i,\pi(i)}TL_2\right)\times\left((1-g_{i-1,\pi(i-1)})P_1+g_{i-1,\pi(i-1)}P_2\right)$ | | | | | | | |
| State 3 | | | | | | ○ | ● | ● |
| | Cost for the *i*th process $C_p^{(2)}\left((1-g_{i,\pi(i)})TL_1+g_{i,\pi(i)}TL_2\right)\times\left((1-g_{i-2,\pi(i-2)})P_1+g_{i-2,\pi(i-2)}P_2\right)\times\left((1-g_{i-1,\pi(i-1)})Q_1+g_{i-1,\pi(i-1)}Q_2\right)$ | | | | | | | |
| … | | | | | | | | |
| State *i* | ○ | ● | ● | … | ● | ● | ● | ● |
| | Cost for the *i*th process $C_p^{(i-1)}\left((1-g_{i,\pi(i)})TL_1+g_{i,\pi(i)}TL_2\right)\times\left((1-g_{1,\pi(1)})P_1+g_{1,\pi(1)}P_2\right)\times\prod_{\beta=2}^{i-1}\left((1-g_{\beta,\pi(\beta)})Q_1+g_{\beta,\pi(\beta)}Q_2\right)$ | | | | | | | |
| State *i* + 1 | ● | ● | ● | … | ● | ● | ● | ● |
| | Cost for the *i*th process $C_p^{(i)}\left((1-g_{i,\pi(i)})TL_1+g_{i,\pi(i)}TL_2\right)\times\prod_{\beta=1}^{i-1}\left((1-g_{\beta,\pi(\beta)})Q_1+g_{\beta,\pi(\beta)}Q_2\right)$ | | | | | | | |

When there are $n$ processes, the cost in the state from the first process to the $n$th process is accumulated. As a sum of these, the expected cost due to idle and delay that occurred is expressed as

$$
\begin{aligned}
&f(n;\pi)\\
&=\sum_{i=1}^{n} C_s\left(\left(1-g_{i,\pi(i)}\right)TS_1+g_{i,\pi(i)}TS_2\right)\\
&\quad+\sum_{i=1}^{n}\begin{pmatrix}\left(1-g_{i,\pi(i)}\right)TL_1\\ +g_{i,\pi(i)}TL_2\end{pmatrix}\left[C_p^{(1)}+\sum_{\alpha=2}^{i}\left(C_p^{(\alpha)}-C_p^{(\alpha-1)}\right)\prod_{\beta=i-\alpha+1}^{i-1}\begin{pmatrix}\left(1-g_{\beta,\pi(\beta)}\right)Q_1\\ +g_{\beta,\pi(\beta)}Q_2\end{pmatrix}\right]\\
&=C_s\left((n-y)TS_1+yTS_2\right)\\
&\quad+\sum_{i=1}^{n}\begin{pmatrix}TL_1(1-g_{i,\pi(i)})(C_p^{(1)}+\sum_{\alpha=2}^{i}(C_p^{(\alpha)}-C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\\ +TL_2 g_{i,\pi(i)}(C_p^{(1)}+\sum_{\alpha=2}^{i}(C_p^{(\alpha)}-C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\end{pmatrix}
\end{aligned}
\tag{3.3.5}
$$

$f(n;\pi)$ and $nC_tZ$ are the two parts of Eq. (3.3.4). Therefore, Theorem 2 was proved.

## 3.4 Problem of optimal assignment

In this section, an assembly line with the reset model is formulated where there are two types of processing capabilities of each worker for each process. Then, theorems of the rule of optimal worker assignment are proposed. The optimal assignment rules are considered in the following two cases. In one of the cases, workers can be assigned to the tasks they are proficient at performing. Next, cases are considered where there is a limited number of processes to assign workers to their inefficient processes. As a first step, so the other is the case where an inefficient worker is assigned to only one task (i.e., proficient workers can be assigned to all the remaining tasks). This section also uses the symbols defined in Section 3.3.2.

### 3.4.1 In the case where workers can be placed for their proficient tasks

A rule of optimal worker assignment when every worker can be placed for their proficient tasks is described. The following symbols are defined.

The assignments when every worker can be placed for their proficient tasks are expressed as $\pi_{(ag)}$. The expected cost of assignment $\pi_{(ag)}$is $f\left(n;\pi_{(ag)}\right)$. The rest of the assignments are expressed as $\pi_{(ng)}$ . The expected cost of assignment $\pi_{(ng)}$ is $f(n;\pi_{(ng)})$. The mathematical explanation of $\pi_{(ag)}$ is as follows.

$G$: A $n \times n$ matrix, where each element is $(1 - g_{ij})$.

$P$: A $n \times n$ permutation matrix.

$Q$: A $n \times n$ permutation matrix.

$\exists P$ and $Q$ , if the trace of the square matrix $PGQ$ is 0, every worker can be assigned to their proficient tasks. In other words, all the elements on the diagonal are 0. The assignments of these cases are expressed as $\pi_{(ag)}$ .

**Theorem 3**

If

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, $TL_1 > TL_2$ and $C_p^{(1)}(TL_1 - TL_2) \geq C_s(TS_2 - TS_1)$

hold, then the optimal assignment is $\pi_{(ag)}$.

In other words, the optimal assignment allows every worker to be assigned to the process they are proficient at performing. This theorem is proved mathematically.

**Proof of Theorem 3**

From Eq. (3.2.3), to prove that the optimal assignment is when every worker is assigned to the process they are proficient at performing, it is sufficient to show that the expected cost $f(n;\pi)$ is the lowest. From Theorem 2, the following equation can be derived:

$$f(n;\pi_{(ag)}) = nC_s TS_2 + TL_2 \sum_{i=1}^{n} (C_p^{(1)} + \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-1}) \qquad (3.4.1)$$

$$\begin{aligned} & f(n;\pi_{(ng)}) \\ & = C_s[(n-y)TS_1 + yTS_2] \\ & + \sum_{i=1}^{n} \left( \begin{aligned} & TL_1(1 - g_{i,\pi(i)})(C_p^{(1)} + \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^{B(i,\alpha)} Q_2^{A(i,\alpha)}) \\ & + TL_2 g_{i,\pi(i)} (C_p^{(1)} + \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^{B(i,\alpha)} Q_2^{A(i,\alpha)}) \end{aligned} \right) \end{aligned} \qquad (3.4.2)$$

According to Eq. (3.2.2) and Eq. (3.2.3),

$$
\begin{aligned}
& TC(n;\pi_{(ng)}) - TC(n;\pi_{(ag)}) \\
& = f(n;\pi_{(ng)}) - f(n;\pi_{(ag)}) \\
& = (n-y)C_s(TS_1 - TS_2) + \sum_{i=1}^{n} C_p^{(1)}(1-g_{i,\pi(i)})(TL_1 - TL_2) \\
& \quad + \sum_{i=1}^{n} \begin{pmatrix} TL_1(1-g_{i,\pi(i)})(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \\ + TL_2 g_{i,\pi(i)}(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \end{pmatrix} \\
& \quad - \sum_{i=1}^{n} \begin{pmatrix} TL_2(1-g_{i,\pi(i)})(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \\ + TL_2 g_{i,\pi(i)}(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \end{pmatrix}
\end{aligned}
\tag{3.4.3}
$$

holds.

If Eq. (3.4.3) is divided into X and Y,

$$
TC(n;\pi_{(ng)}) - TC(n;\pi_{(ag)}) = X + Y \tag{3.4.4}
$$

here,

$$
\begin{aligned}
Y & \equiv (n-y)C_s(TS_1 - TS_2) + \sum_{i=1}^{n} C_p^{(1)}(1-g_{i,\pi(i)})(TL_1 - TL_2) \\
& = (n-y)C_s(TS_1 - TS_2) + (n-y)C_p^{(1)}(TL_1 - TL_2)
\end{aligned}
\tag{3.4.5}
$$

$$
\begin{aligned}
X \equiv & \sum_{i=1}^{n} \begin{pmatrix} TL_1(1-g_{i,\pi(i)})(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \\ + TL_2 g_{i,\pi(i)}(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \end{pmatrix} \\
& - \sum_{i=1}^{n} \begin{pmatrix} TL_2(1-g_{i,\pi(i)})(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \\ + TL_2 g_{i,\pi(i)}(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{B(i,\alpha)}Q_2^{A(i,\alpha)}) \end{pmatrix}
\end{aligned}
\tag{3.4.6}
$$

If $Q_1 > Q_2$ and $TL_1 > TL_2$, then $X > 0$. If $C_p^{(1)}(TL_1 - TL_2) \geq C_s(TS_2 - TS_1)$, then $Y > 0$. Therefore, Eq. (3.4.3) is positive.

Theorem 3 was proved.

### 3.4.2 In the case where an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all the remaining tasks

A rule of optimal worker assignment when an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all the remaining tasks is described. Before considering the problem of optimal assignment, the meaning that there is a limited number of processes to assign workers to their inefficient processes is explained mathematically. The following symbols are defined.

For $i = 1,2\cdots n$ and $j = 1,2\cdots n$,

$N$ : $N = \{1,2,3,\cdots n\}$

$R$ : A set of processes that can be assigned by either proficient or inefficient workers. $R = \{r_1, r_2, r_3, \cdots r_m\}$, $1 \le m \le n$, $1 \le r_1 < r_2 < \cdots < r_m \le n$. In addition, it is possible to assign inefficient workers to the processes in the set and to assign proficient workers to other processes in the set.

$G$ : A set of processes that can be assigned only by proficient workers. $G = R - N$, $G \cap R = \phi$.

The meaning of sets $R$ and $G$ is explained in the following example. In Figure 3.5, ★ indicates a process that can be assigned only by proficient workers, and △ indicates a process that can be assigned by either proficient or inefficient workers. That is, the process represented by ★ is an element in the set, and the process represented by △ is an element in the set. According to $R = \{r_1, r_2, r_3, \cdots r_m\}$, the $r_1$ th process is the first process among the processes where inefficient workers can be assigned, and the $r_m$ th process is the last process among the processes where inefficient workers can be assigned.

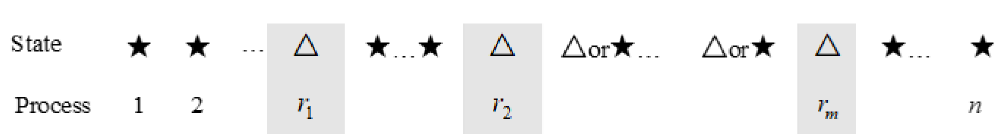


Figure 3.5: Image of the set *R*.

A case is considered where an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all the remaining tasks. The meaning that inefficient worker is assigned to only one task means that if an inefficient worker is assigned to any one of the set $R$, a proficient worker can be positioned at the remaining points of the processes.

Assignment $\pi$ is defined at the beginning of Section 3.2.3. An assignment in which inefficient workers are positioned at the process $r$ $(r \in R)$ and all proficient workers are positioned at the remaining points of the processes is represented by $\pi = \varphi[r]$.

Before the theorem, the following lemmas are declared, which are useful for the proof of the theorem.

**Lemma**

Let $\beta, \gamma \in R$, $\beta < \gamma$

(1) If $\beta + \gamma < n$,

$$\begin{aligned} &f(n;\varphi[\gamma]) - f(n;\varphi[\beta]) \\ &= \sum_{\alpha=\beta+1}^{\gamma} (TL_1 - TL_2)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-1} \\ &\quad + TL_2 \sum_{i=\gamma+\beta+1}^{n} \sum_{\alpha=i-\gamma+1}^{i-\beta} (C_p^{(\alpha)} - C_p^{(\alpha-1)})(Q_1 - Q_2)Q_2^{\alpha-2}. \end{aligned} \tag{3.4.7}$$

(2) If $\beta + \gamma = n$,

$$f(n;\varphi[\gamma]) - f(n;\varphi[\beta]) = \sum_{\alpha=\beta+1}^{\gamma} (TL_1 - TL_2)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-1}. \tag{3.4.8}$$

(3) If $\beta + \gamma = n+1$,

$$f(n;\varphi[\gamma]) - f(n;\varphi[\beta]) = \sum_{\alpha=\beta+1}^{\gamma} (TL_1Q_2 - TL_2Q_1)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}. \tag{3.4.9}$$

(4) If $2 + n \le \beta + \gamma < 2n$,

$$\begin{aligned} &f(n;\varphi[\gamma]) - f(n;\varphi[\beta]) \\ &= TL_2(Q_2 - Q_1) \sum_{i=n}^{\beta+\gamma-2} \sum_{\alpha=i-\gamma+2}^{i-\beta+1} (C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2} \\ &\quad + (TL_1Q_2 - TL_2Q_1) \sum_{\alpha=\beta+1}^{\gamma} (C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}. \end{aligned} \tag{3.4.10}$$

**Proof of Lemma**

For $i = 1, 2, \ldots n$, the sum of the expected idle cost and the expected delay cost $f(n;\pi)$

can be expressed as

$$
\begin{aligned}
&f(n;\pi)\\
&= C_s\big((n-y)TS_1 + yTS_2\big)\\
&\quad + \sum_{i=1}^{n}\left(\begin{aligned}&TL_1(1-g_{i,\pi(i)})(C_p^{(1)} + \sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\\ &+ TL_2 g_{i,\pi(i)}(C_p^{(1)} + \sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\end{aligned}\right)
\end{aligned}
\tag{3.4.11}
$$

Considering a case where an inefficient worker is assigned to only one task (i.e., proficient workers can be assigned to all the remaining tasks), the number of workers who work on the proficient tasks in the assignment $\pi = \varphi[r]$ is $n-1$. If $y = n-1$ is substituted into Eq. (3.4.11), the following equation can be derived:

$$
\begin{aligned}
&f(n;\varphi[r])\\
&= C_s\big(TS_1 + (n-1)TS_2\big) + C_p^{(1)}\big(TL_1 + (n-1)TL_2\big)\\
&\quad + \sum_{i=1}^{n}\left(\begin{aligned}&TL_1(1-g_{i,\pi(i)})(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\\ &+ TL_2 g_{i,\pi(i)}(\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{B(i,\alpha)}Q_2^{A(i,\alpha)})\end{aligned}\right)
\end{aligned}
\tag{3.4.12}
$$

When $i = r$, the expected delay time is $TL_1$. $B(i,\alpha) = 0$. $A(i,\alpha) = \alpha - 1$.

When $i < r$, the expected delay time is $TL_2$. $B(i,\alpha) = 0$. $A(i,\alpha) = \alpha - 1$.

When $i > r$, the expected delay time is $TL_2$. If $i - \alpha < r$, then $B(i,\alpha) = 1$, $A(i,\alpha) = \alpha - 2$. If $i - \alpha \geq r$, then $B(i,\alpha) = 0$, $A(i,\alpha) = \alpha - 1$.

Thus, the following equation can be derived:

$$
\begin{aligned}
&f(n;\varphi[r])\\
&= C_s\big(TS_1 + (n-1)TS_2\big) + C_p^{(1)}\big(TL_1 + (n-1)TL_2\big)\\
&\quad + \sum_{i=1}^{r-1}\left(TL_2\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{0}Q_2^{\alpha-1}\right)\\
&\quad + \sum_{i=r}^{r}\left(TL_1\sum_{\alpha=2}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{0}Q_2^{\alpha-1}\right)\\
&\quad + \sum_{i=r+1}^{n}\left(\begin{aligned}&TL_2\sum_{\alpha=2}^{i-r}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{0}Q_2^{\alpha-1}\\ &+ TL_2\sum_{\alpha=i-r+1}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^{1}Q_2^{\alpha-2}\end{aligned}\right)
\end{aligned}
\tag{3.4.13}
$$

For $\beta, \gamma \in R$, $\beta < \gamma$

$$f(n;\varphi[\gamma]) - f(n;\varphi[\beta])$$

$$= \left( \begin{array}{l} \sum_{i=1}^{\gamma-1} \left( TL_2 \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) + \sum_{i=\gamma}^{\gamma} \left( TL_1 \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) \\ + \sum_{i=\gamma+1}^{n} \left( TL_2 \sum_{\alpha=2}^{i-\gamma} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} + TL_2 \sum_{\alpha=i-\gamma+1}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^1 Q_2^{\alpha-2} \right) \end{array} \right)$$

$$- \left( \begin{array}{l} \sum_{i=1}^{\beta-1} \left( TL_2 \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) + \sum_{i=\beta}^{\beta} \left( TL_1 \sum_{\alpha=2}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) \\ + \sum_{i=\beta+1}^{n} \left( TL_2 \sum_{\alpha=2}^{i-\beta} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} + TL_2 \sum_{\alpha=i-\beta+1}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^1 Q_2^{\alpha-2} \right) \end{array} \right) . \tag{3.4.14}$$

It is divided into 4 cases to prove Lemma. (1) $\beta + \gamma < n$, (2) $\beta + \gamma = n$, (3) $\beta + \gamma = n+1$ and (4) $2 + n \le \beta + \gamma < 2n$

(1) When $\beta + \gamma < n$,

Eq. (3.4.14) can be transformed into

$$f(n;\varphi[\gamma]) - f(n;\varphi[\beta])$$

$$= \left( \begin{array}{l} \sum_{i=\beta}^{\gamma-1} \left( TL_2 \sum_{\alpha=i-\beta+1}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) \\ + \sum_{i=\gamma+1}^{\gamma+\beta} \left( TL_2 \sum_{\alpha=i-\gamma+1}^{i-\beta} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^1 Q_2^{\alpha-2} \right) \end{array} \right)$$

$$- \left( \begin{array}{l} \sum_{i=\beta+1}^{\gamma} \left( TL_2 \sum_{\alpha=i-\beta+1}^{i} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^1 Q_2^{\alpha-2} \right) \\ + \sum_{i=\gamma}^{\gamma+\beta} \left( TL_2 \sum_{\alpha=i-\gamma+1}^{i-\beta} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1} \right) \end{array} \right) \tag{3.4.15}$$

$$+ TL_1 \sum_{\alpha=\beta+1}^{\gamma} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_1^0 Q_2^{\alpha-1}$$

$$+ \sum_{i=\gamma+\beta+1}^{n} \left( TL_2 \sum_{\alpha=i-\gamma+1}^{i-\beta} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) (Q_1 - Q_2) Q_2^{\alpha-2} \right)$$

Next, from

$$\sum_{i=\gamma}^{\beta+\gamma}\left( TL_2 \sum_{\alpha=i-\gamma+1}^{i-\beta}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^0 Q_2^{\alpha-1} \right)$$
$$-\sum_{i=\beta}^{\gamma-1}\left( TL_2 \sum_{\alpha=i-\beta+1}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^0 Q_2^{\alpha-1} \right) \quad (3.4.16)$$
$$= \sum_{\alpha=\beta+1}^{\gamma} TL_2 (C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^0 Q_2^{\alpha-1}$$

and

$$\sum_{i=\beta+1}^{\gamma}\left( TL_2 \sum_{\alpha=i-\beta+1}^{i}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^1 Q_2^{\alpha-2} \right)$$
$$= \sum_{i=\gamma+1}^{\beta+\gamma}\left( TL_2 \sum_{\alpha=i-\gamma+1}^{i-\beta}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_1^1 Q_2^{\alpha-2} \right) \quad (3.4.17)$$

the following equation can be derived:

$$f(n;\varphi[\gamma]) - f(n;\varphi[\beta])$$
$$= \sum_{\alpha=\beta+1}^{\gamma}(TL_1 - TL_2)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-1} \quad (3.4.18)$$
$$+ TL_2 \sum_{i=\beta+\gamma+1}^{n} \sum_{\alpha=i-\gamma+1}^{i-\beta}(C_p^{(\alpha)} - C_p^{(\alpha-1)})(Q_1 - Q_2)Q_2^{\alpha-2}$$

Therefore, Lemma (1) was proved.

Proof of Lemma (2), Lemma (3) and Lemma (4) can be proved by the same consideration of Lemma (1).

Using the lemma above, the theorem of the optimal assignment is obtained as follows:

**Theorem 4**

(a) The case of $r_m \le n/2$

If

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, and $TL_1 > TL_2$

hold, then the optimal assignment is $\pi = \varphi[r_1]$.

(b) The case of $r_m > n/2$

(b-1) When $r_1 < n - r_m$ ,

if

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, and $TL_1 > TL_2$

hold, then the optimal assignment is $\pi = \varphi[r_1]$.

(b-2) When $r_1 = n - r_m$ ,

if

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, and $TL_1 > TL_2$

hold, then the optimal assignment is $\pi = \varphi[r_1]$.

(b-3) When $r_1 = n - r_m + 1$ ,

if

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, $TL_1 > TL_2$ and $TL_1 / Q_1 > TL_2 / Q_2$

hold, then the optimal assignment is $\pi = \varphi[r_1]$.

(b-4) When $n - r_m + 2 \le r_1 < 2n - r_m$,

(b-4-1)

if

1) $C_P^{(\alpha)}$ is increasing in α,

2) $Q_1 > Q_2$, $TL_1 > TL_2$ and

$$(TL_1Q_2 - TL_2Q_1)\sum_{\alpha=r_1+1}^{r}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

$$> TL_2(Q_1 - Q_2)\sum_{i=n}^{r_1+r-2}\sum_{\alpha=i-r+2}^{i-r_1+1}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

hold, then the optimal assignment is $\pi = \varphi[r_1]$.

(b-4-2)

if

1) $C_P^{(\alpha)}$ is increasing in α,

2) $TL_1 / Q_1 > TL_2 / Q_2$ and

$$(TL_1 Q_2 - TL_2 Q_1) \sum_{\alpha=r+1}^{r_m} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2}$$

$$< TL_2 (Q_1 - Q_2) \sum_{i=n}^{r+r_m-2} \sum_{\alpha=i-r_m+2}^{i-r+1} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2}$$

hold, then the optimal assignment is $\pi = \varphi[r_m]$.

Next is the explanation of Theorem 4. A rule of optimal worker assignment when an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all the remaining tasks is described in Theorem 4. The meanings of the symbols have been defined in the previous texts and are repeated here for ease of explanation.

$\pi$ represents the assignment of the workers. One production will be processed by $n$ processes.

$R$ is a set of processes that can be assigned by either proficient or inefficient workers. $R = \{r_1, r_2, r_3, \cdots r_m\}$, $1 \le m \le n$, $1 \le r_1 < r_2 < \cdots < r_m \le n$.

An assignment in which inefficient workers are positioned at the process $r$ ($r \in R$) and all proficient workers are positioned at the remaining points of the processes is represented by $\pi = \varphi[r]$.

Theorem 4 (a) shows the case where the element $r_m$ is before or equal to $n/2$. If $Q_1 > Q_2$ and $TL_1 > TL_2$, the optimal worker assignment is $\pi = \varphi[r_1]$. Here, $Q_1 > Q_2$ represents the probability of an inefficient worker becoming delayed is higher than a proficient worker. $TL_1 > TL_2$ represents the expected delay time of an inefficient worker is higher than a proficient worker. The above inequalities represent a worker with proficient capabilities can complete work in the processes according to a standard schedule while a worker with inefficient capabilities needs more time than the standard scheduled time to complete the work of the processes. $\pi = \varphi[r_1]$ represents the assignment in which inefficient workers are positioned at the process $r_1$ and all proficient workers are positioned at the remaining points of the processes. Theorem 4 (a) states that to minimize the expected cost due to idle and delay, inefficient workers should be placed at the starting point of the process that can assign the inefficient workers, and proficient workers should be positioned at the remaining points of the process. The same meaning is omitted in the following description.

Theorem 4 (b) shows the case where the element $r_m$ is after $n/2$.

(b-1) discusses the case where the numerical magnitude of the sum of the element $r_1$ and the element $r_m$ is less than $n$. If $Q_1 > Q_2$ and $TL_1 > TL_2$, the optimal worker assignment is $\pi = \varphi[r_1]$.

(b-2) discusses the case where the numerical magnitude of the sum of the element $r_1$ and the element $r_m$ is equal to $n$. If $Q_1 > Q_2$ and $TL_1 > TL_2$, the optimal worker assignment is $\pi = \varphi[r_1]$.

(b-3) discusses the case where the numerical magnitude of the sum of the element $r_1$ and the element $r_m$ is equal to $n+1$. If $Q_1 > Q_2$, $TL_1 > TL_2$ and $TL_1/Q_1 > TL_2/Q_2$, the optimal worker assignment is $\pi = \varphi[r_1]$. Here, $TL_1/Q_1$ represents under the condition that the inefficient worker's working time is longer than Z, the conditional expected value of the difference between the inefficient worker's working time and Z. Therefore, the inequalities of $TL_1/Q_1 > TL_2/Q_2$ also represents a worker with proficient capabilities can complete work in the processes according to a standard schedule while a worker with inefficient capabilities needs more time than the standard scheduled time to complete the work of the processes.

(b-4) discusses the case where the numerical magnitude of the sum of the element $r_1$ and the element $r_m$ is from $n+2$, and less than $2n$.

(b-4-1) If $Q_1 > Q_2$ and $TL_1 > TL_2$ hold, and the inequality of

$$(TL_1Q_2 - TL_2Q_1)\sum_{\alpha=r_1+1}^{r}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

$$> TL_2(Q_1 - Q_2)\sum_{i=n}^{r_1+r-2}\sum_{\alpha=i-r+2}^{i-r_1+1}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

numerically satisfies, the optimal worker assignment is $\pi = \varphi[r_1]$.

(b-4-2) If $TL_1/Q_1 > TL_2/Q_2$ holds, and the inequality of

$$(TL_1Q_2 - TL_2Q_1)\sum_{\alpha=r+1}^{r_m}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

$$< TL_2(Q_1 - Q_2)\sum_{i=n}^{r+r_m-2}\sum_{\alpha=i-r_m+2}^{i-r+1}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}$$

numerically satisfies, the optimal worker assignment is $\pi = \varphi[r_m]$. Here, $\pi = \varphi[r_m]$ represents the assignment in which inefficient workers are positioned at the process $r_m$ and all proficient workers are positioned at the remaining points of the processes. Theorem 4 (b-4-2) states that to minimize the expected cost due to idle and delay, inefficient workers should be placed at the ending point of the process that can assign the inefficient workers, and proficient workers should be positioned at the remaining points of the process.

**Proof of Theorem 4**

The proof is described below.

(a) The case of $r_m \le n/2$

If $\beta = r_1$ and $\gamma \in \{r_2, r_3, \cdots r_m\}$, then $\beta + \gamma \le r_1 + r_m < n$. The condition of Lemma (1) holds. From Eq. (3.4.7), the following equation can be derived:

$$\begin{aligned} & f(n;\varphi[r]) - f(n;\varphi[r_1]) \\ &= \sum_{\alpha=r_1+1}^{r} (TL_1 - TL_2)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-1} \\ &\quad + TL_2 \sum_{i=r+r_1+1}^{n} \sum_{\alpha=i-r+1}^{i-r_1} (C_p^{(\alpha)} - C_p^{(\alpha-1)})(Q_1 - Q_2)Q_2^{\alpha-2}. \end{aligned} \tag{3.4.19}$$

If $C_P^{(\alpha)}$ is increasing in $\alpha$, $Q_1 > Q_2$, and $TL_1 > TL_2$, Eq. (3.4.19) is positive. Therefore, among the processes in which each of the inefficient workers can be assigned, the assignment (that is, $\varphi[r_1]$) in which the inefficient worker is placed at the starting point of the process and the proficient worker is positioned at the remaining points of the process is performed. This is the optimal assignment.

(b) The case of $r_m > n/2$

(b-1) When $r_1 < n - r_m$,

if $\beta = r_1$ and $\gamma \in \{r_2, r_3, \cdots r_m\}$, then $\beta + \gamma \le r_1 + r_m < n$. The condition of Lemma (1) holds. From Eq. (3.4.7), Eq. (3.4.19) can be derived. If $C_P^{(\alpha)}$ is increasing in $\alpha$, $Q_1 > Q_2$, and $TL_1 > TL_2$, Eq. (3.4.19) is positive. Therefore, the optimal assignment is $\varphi[r_1]$.

(b-2) When $r_1 = n - r_m$,

if $\beta = r_1$ and $\gamma \in \{r_2, r_3, \cdots r_m\}$, then $\beta + \gamma \le r_1 + r_m \le n$. When $\beta + \gamma < n$, the condition of Lemma (1) holds. From Eq. (3.4.7), Eq. (3.4.19) can be derived. When $\beta + \gamma = n$, the condition of Lemma (2) holds. From Eq. (3.4.8), the following equation can be derived:

$$f(n;\varphi[r]) - f(n;\varphi[r_1]) = \sum_{\alpha=r_1+1}^{r} (TL_1 - TL_2)(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-1}. \tag{3.4.20}$$

If $C_P^{(\alpha)}$ is increasing in $\alpha$, $Q_1 > Q_2$, and $TL_1 > TL_2$, Eq. (3.4.19) and Eq. (3.4.20) are positive. Therefore, the optimal assignment is $\varphi[r_1]$.

(b-3) When $r_1 = n - r_m + 1$ ,

if $\beta = r_1$ and $\gamma \in \{r_2, r_3, \cdots r_m\}$, then $\beta + \gamma \le r_1 + r_m \le n+1$. When $\beta + \gamma < n$, the condition of Lemma (1) holds. From Eq. (3.4.7), Eq. (3.4.19) can be derived. When $\beta + \gamma = n$, the condition of Lemma (2) holds. From Eq. (3.4.8), Eq. (3.4.20) can be derived. When $\beta + \gamma = n+1$, the condition of Lemma (3) holds. From Eq. (3.4.9), the following equation can be derived:

$$f(n;\varphi[r]) - f(n;\varphi[r_1]) = \sum_{\alpha=r_1+1}^{r} (TL_1 Q_2 - TL_2 Q_1)(C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2} . \qquad (3.4.21)$$

If $C_P^{(\alpha)}$ is increasing in $\alpha$ , $Q_1 > Q_2$, $TL_1 > TL_2$ and $TL_1 / Q_1 > TL_2 / Q_2$, Eq. (3.4.19), Eq. (3.4.20) and Eq. (3.4.21) are positive. Therefore, the optimal assignment is $\varphi[r_1]$.

(b-4) When $n - r_m + 2 \le r_1 < 2n - r_m$,

(b-4-1) if $\beta = r_1$ and $\gamma \in \{r_2, r_3, \cdots r_m\}$, then $r_1 + r_2 \le \beta + \gamma \le r_1 + r_m$. When $\beta + \gamma < n$, the condition of Lemma (1) holds. From Eq. (3.4.7), Eq. (3.4.19) can be derived. When $\beta + \gamma = n$, the condition of Lemma (2) holds. From Eq. (3.4.8), Eq. (3.4.20) can be derived. When $\beta + \gamma = n+1$, the condition of Lemma (3) holds. From Eq. (3.4.9), Eq. (3.4.21) can be derived. When $n+2 \le \beta + \gamma < 2n$, the condition of Lemma (4) holds. From Eq. (3.4.10), the following equation can be derived:

$$\begin{aligned} & f(n;\varphi[r]) - f(n;\varphi[r_1]) \\ & = TL_2 (Q_2 - Q_1) \sum_{i=n}^{r_1+r-2} \sum_{\alpha=i-r+2}^{i-r_1+1} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2} \\ & \quad + (TL_1 Q_2 - TL_2 Q_1) \sum_{\alpha=r_1+1}^{r} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2}. \end{aligned} \qquad (3.4.22)$$

If $C_P^{(\alpha)}$ is increasing in $\alpha$ , $Q_1 > Q_2$, $TL_1 > TL_2$ and

$$\begin{aligned} & (TL_1 Q_2 - TL_2 Q_1) \sum_{\alpha=r_1+1}^{r} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2} \\ & > TL_2 (Q_1 - Q_2) \sum_{i=n}^{r_1+r-2} \sum_{\alpha=i-r+2}^{i-r_1+1} (C_p^{(\alpha)} - C_p^{(\alpha-1)}) Q_2^{\alpha-2} \end{aligned}$$

Eq. (3.4.19), Eq. (3.4.20), Eq. (3.4.21) and Eq. (3.4.22) are positive. Therefore, among the processes in which each of the inefficient workers can be assigned, the assignment (that is, $\varphi[r_1]$) in which the inefficient worker is placed at the starting point of the process and the proficient worker is positioned at the remaining points of the process is performed. This is the optimal assignment.

(b-4-2) if $\beta \in \{r_1, r_2, \cdots r_{m-1}\}$ and $\gamma = r_m$, then $\beta + \gamma \geq r_1 + r_m$. When $n+2 \leq \beta + \gamma < 2n$, the condition of Lemma (4) holds. From Eq. (3.4.10), the following equation can be derived:

$$\begin{aligned} & f(n;\varphi[r_m]) - f(n;\varphi[r]) \\ & = TL_2(Q_2 - Q_1)\sum_{i=n}^{r+r_m-2}\sum_{\alpha=i-r_m+2}^{i-r+1}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2} \\ & \quad + (TL_1Q_2 - TL_2Q_1)\sum_{\alpha=r+1}^{r_m}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2}. \end{aligned} \tag{3.4.23}$$

If $C_P^{(\alpha)}$ is increasing in $\alpha$, $TL_1 / Q_1 > TL_2 / Q_2$ and

$$\begin{aligned} & (TL_1Q_2 - TL_2Q_1)\sum_{\alpha=r+1}^{r_m}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2} \\ & < TL_2(Q_1 - Q_2)\sum_{i=n}^{r+r_m-2}\sum_{\alpha=i-r_m+2}^{i-r+1}(C_p^{(\alpha)} - C_p^{(\alpha-1)})Q_2^{\alpha-2} \end{aligned}$$

Eq. (3.4.23) is negative. Therefore, among the processes in which each of the inefficient workers can be assigned, the assignment (that is, $\varphi[r_m]$) in which the inefficient worker is placed at the ending point of the process and the proficient worker is positioned at the remaining points of the process is performed. This is the optimal assignment.

Therefore, Theorem 4 was proved.

**Numerical experiments**

The validity of Theorem 4 is confirmed through numerical experiments. Although the above theorem is always holding no matter what distribution the working time follows, a certain distribution needs to be assumed to prove it through a numeral experiment. It is supposed that the working time of workers follows exponential distribution. For process $i$, the working time probability density function of worker $j$ is $f_{ij}(t) = \mu_{ij} e^{-\mu_{ij} t}$

$$P_{ij} = \int_0^Z f_{ij}(t)dt = 1 - e^{-\mu_{ij} Z}$$

$$Q_{ij} = \int_Z^\infty f_{ij}(t)dt = e^{-\mu_{ij} Z}$$

$$TS_{ij} = \int_0^Z (Z - t) f_{ij}(t)dt = Z - \frac{1}{\mu_{ij}}\left(1 - e^{-\mu_{ij} Z}\right)$$

$$TL_{ij} = \int_Z^{\infty} (t - Z) f_{ij}(t) dt = \frac{1}{\mu_{ij}} e^{-\mu_{ij} Z} .$$

Where $\mu_{ij}$ is the processing rate of worker $j$ assigned to process $i$ .

$$\mu_{ij} = \begin{cases} \mu_1 & \text{When worker } j \text{ is inefficient at process } i \\ \mu_2 & \text{When worker } j \text{ is proficient at process } i \end{cases} \qquad (\mu_1 > \mu_2)$$

Other parameters are assumed as follows,

| | | |
|---|---|---|
| The number of processes | : | $n = 9, 10$ |
| Target working time | : | $Z = 2$ |
| Target processing cost | : | $C_t = 10$ |
| Idle cost | : | $C_S = 20$ |
| Delay cost | : | $C_P^{(1)} = 50$ $C_P^{(2)} = 70$ $C_P^{(3)} = 90$ $C_P^{(4)} = 110$ $C_P^{(5)} = 130$ $C_P^{(6)} = 150$ $C_P^{(7)} = 170$ $C_P^{(8)} = 190$ $C_P^{(9)} = 210$ $C_P^{(10)} = 230$ |

Specifically, the processing rate of each worker changes in the following two cases. In one of the cases, numerical experiments are performed when the proficient processing rate is fixed at 1.0 and the inefficient processing varied from 0.1 to 0.9. In the other case, numerical experiments are performed when the inefficient processing rate is fixed at 0.1 and the proficient processing varied from 0.2 to 1.0.

The results of numerical experiments are shown here as an example for the case when the proficient processing rate is fixed and the inefficient processing is changed on the *n*=9 system. The total expected cost is shown in Table 3.3. As an example, the total expected cost $TC(9;\varphi[2])$ is 473.0653809, when the proficient processing rate is 1.0 and the inefficient processing is 0.5. Comparing the total expected cost, the optimal assignment can be found in Table 3.3. For example, when $r_1 = 2, r_2 = 3$ and $r_m = r_3 = 7$, $TC(9,\varphi[2]) < TC(9,\varphi[3]) < TC(9,\varphi[7])$ holds. the optimal assignment is $\varphi[2]$ because the total expected cost of assignment $\varphi[2]$ is the lowest.

The optimal assignment is shown in Table 3.4~3.8. For example, in Table 3.5, when $r_1 = 2$ and $r_m = 5$, the set of $R$ can be $R = \{2,5\}$, $R = \{2,3,5\}$, $R = \{2,4,5\}$ or $R = \{2,3,4,5\}$, the optimal assignment for the above cases is $\varphi[2]$. It can be seen that Theorem 4 (b-1) is correct.

From Table 3.4~3.8, it can be correspondingly gotten that Theorem 4 (a), (b-1), (b-2), (b-3), and (b-4) are all correct.

Table 3.3: The total expected cost when the proficient processing rate is fixed and the inefficient processing is varied ($n$=9).

| $\mu_1$ | $\mu_2$ | $TC(9;\varphi[1])$ | $TC(9;\varphi[2])$ | $TC(9;\varphi[3])$ | $TC(9;\varphi[4])$ | $TC(9;\varphi[5])$ |
|---|---|---|---|---|---|---|
| 0.1 | 1.0 | 834.3614238 | 856.4452594 | 859.4339687 | 859.8383512 | 859.8923739 |
| 0.2 | 1.0 | 595.3973349 | 604.3294608 | 605.5382825 | 605.7018041 | 605.7233829 |
| 0.3 | 1.0 | 521.7943364 | 526.5547625 | 527.1990083 | 527.2861398 | 527.2975055 |
| 0.4 | 1.0 | 488.7261075 | 491.5333196 | 491.9132285 | 491.9645998 | 491.9712284 |
| 0.5 | 1.0 | 471.3416948 | 473.0653809 | 473.2986520 | 473.3301894 | 473.3342178 |
| 0.6 | 1.0 | 461.4358162 | 462.4985088 | 462.6423254 | 462.6617658 | 462.6642258 |
| 0.7 | 1.0 | 455.5471840 | 456.1815286 | 456.2673757 | 456.2789783 | 456.2804339 |
| 0.8 | 1.0 | 451.9849333 | 452.3299119 | 452.3765984 | 452.3829075 | 452.3836927 |
| 0.9 | 1.0 | 449.8387825 | 449.9822919 | 450.0017132 | 450.0043374 | 450.0046617 |

| $\mu_1$ | $\mu_2$ | $TC(9;\varphi[6])$ | $TC(9;\varphi[7])$ | $TC(9;\varphi[8])$ | $TC(9;\varphi[9])$ |
|---|---|---|---|---|---|
| 0.1 | 1.0 | 859.8944795 | 859.8563000 | 859.5669171 | 857.4276665 |
| 0.2 | 1.0 | 605.7222281 | 605.6919606 | 605.4653707 | 603.7906870 |
| 0.3 | 1.0 | 527.2958942 | 527.2724039 | 527.0972650 | 525.8029421 |
| 0.4 | 1.0 | 491.9697337 | 491.9518586 | 491.8188533 | 490.8359446 |
| 0.5 | 1.0 | 473.3329916 | 473.3197371 | 473.2212310 | 472.4932872 |
| 0.6 | 1.0 | 462.6632953 | 462.6538341 | 462.5835750 | 462.0643789 |
| 0.7 | 1.0 | 456.2797834 | 456.2734330 | 456.2263012 | 455.8780135 |
| 0.8 | 1.0 | 452.3832920 | 452.3794914 | 452.3512950 | 452.1429352 |
| 0.9 | 1.0 | 450.0044773 | 450.0027659 | 449.9900728 | 449.8962763 |

Table 3.4: The optimal assignment when $\mu_1$ is changing ($r_m \le n/2$, $n$=9).

| $\mu_1$ | $\mu_2$ | $r_1$ 1, $r_m$ 2 | $r_1$ 2, $r_m$ 3 | $r_1$ 3, $r_m$ 4 | $r_1$ 1, $r_m$ 3 | $r_1$ 2, $r_m$ 4 | $r_1$ 1, $r_m$ 4 |
|---|---|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.2 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.3 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.4 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.5 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.6 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.7 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.8 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.9 | 1.0 | $\varphi[1]$ | $\varphi[2]$ | $\varphi[3]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ |

Table 3.5: The optimal assignment when $\mu_1$ is changing ($r_m > n/2$ and $r_1 < n - r_m$, $n$=9).

| $\mu_1$ | $\mu_2$ | $r_1$ 3, $r_m$ 5 | $r_1$ 2, $r_m$ 5 | $r_1$ 1, $r_m$ 5 | $r_1$ 2, $r_m$ 6 | $r_1$ 1, $r_m$ 6 | $r_1$ 1, $r_m$ 7 |
|---|---|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.2 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.3 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.4 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.5 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.6 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.7 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.8 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |
| 0.9 | 1.0 | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[2]$ | $\varphi[1]$ | $\varphi[1]$ |

Table 3.6: The optimal assignment when $\mu_1$ is changing ( $r_m > n/2$ and $r_1 = n - r_m$ , $n$=9).

| $\mu_1$ | $\mu_2$ | $r_1$ 4, $r_m$ 5 | $r_1$ 3, $r_m$ 6 | $r_1$ 2, $r_m$ 7 | $r_1$ 1, $r_m$ 8 |
|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.2 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.3 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.4 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.5 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.6 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.7 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.8 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.9 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |

Table 3.7: The optimal assignment when $\mu_1$ is changing ( $r_m > n/2$ and $r_1 = n - r_m + 1$, $n$=9).

| $\mu_1$ | $\mu_2$ | $r_1$ 4, $r_m$ 6 | $r_1$ 3, $r_m$ 7 | $r_1$ 2, $r_m$ 8 | $r_1$ 1, $r_m$ 9 |
|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.2 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.3 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.4 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.5 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.6 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.7 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.8 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |
| 0.9 | 1.0 | $\varphi[4]$ | $\varphi[3]$ | $\varphi[2]$ | $\varphi[1]$ |

Table 3.8: The optimal assignment when $\mu_1$ is changing ( $r_m > n/2$ and $n - r_m + 2 \le r_1 < 2n - r_m$, $n$=9).

| $\mu_1$ | $\mu_2$ | $r_1$ 5, $r_m$ 6 | $r_1$ 6, $r_m$ 7 | $r_1$ 7, $r_m$ 8 | $r_1$ 8, $r_m$ 9 | $r_1$ 5, $r_m$ 7 | $r_1$ 6, $r_m$ 8 | $r_1$ 7, $r_m$ 9 | $r_1$ 4, $r_m$ 7 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[5]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[4]$ |
| 0.2 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.3 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.4 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.5 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.6 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.7 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.8 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |
| 0.9 | 1.0 | $\varphi[6]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[7]$ |

| $\mu_1$ | $\mu_2$ | $r_1$ 5, $r_m$ 8 | $r_1$ 6, $r_m$ 9 | $r_1$ 4, $r_m$ 8 | $r_1$ 5, $r_m$ 9 | $r_1$ 3, $r_m$ 8 | $r_1$ 4, $r_m$ 9 | $r_1$ 3, $r_m$ 9 | $r_1$ 2, $r_m$ 9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[3]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[2]$ |
| 0.2 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.3 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.4 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.5 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.6 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.7 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.8 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |
| 0.9 | 1.0 | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[8]$ | $\varphi[9]$ | $\varphi[9]$ | $\varphi[9]$ |

As the article space constraints, other results of the numerical experiment are omitted. Through all the numerical experiments, the validity of Theorem 4 is confirmed.

## 3.5 Optimal assignment of workers on staffing

For the reset model, the rule of optimal assignment that minimizes the total expected cost is theoretically derived by classifying workers according to their work performance. When considering the operation of the rule of optimal assignment, it is necessary to redo the assignment because the work performance gradually changes with the passage of time from the time of the first optimal assignment as the worker becomes more proficient. Therefore, a method is proposed to adjust workers' assignments according to the difference between the planned and measured values by measuring the working time of each process.

First, the optimal assignment of workers is planned. Next, focusing on the change of the worker for each cycle time, the working time of the worker is measured sequentially. Furthermore, the next working time is predicted based on the measurement results. By predicting the classification of workers for the next in advance, the assignment of workers is adjusted in a timely manner to achieve the optimum.

Generally, a regression model may be used to predict working time. Here is an example of using an autoregressive model to predict the next working time.

An autoregressive model is used to predict the next working time based on historical data. The work time prediction model is expressed by Eq. (3.5.1)[99].

$$X_t = \sum_{i=1}^{p} a_i X_{t-i} + \varepsilon_t \tag{3.5.1}$$

Here, $X_t$ is the working time when the worker performed this work for the *t*-th time. $a_i$, $p$, and $\varepsilon_t$ are the autoregressive coefficient, order of the autoregressive model, and error, respectively. The working time of the *t*-th time is the predicted result, and the working time of the 1st to (*t*-1) th time is the measured value.

That is, a worker's next working time is explained by the past working time multiplied by the autoregressive coefficient $a_i$ and the error $\varepsilon_t$. Furthermore, a threshold value is set for the working time estimated by Eq. (3.5.1), so that proficient and inefficient workers can be discriminated.

When the worker's classification changes based on the discrimination results, the worker is reassigned. Figure 3.6 shows a schematic diagram of the operation of the rule of optimal assignment based on the above procedure.

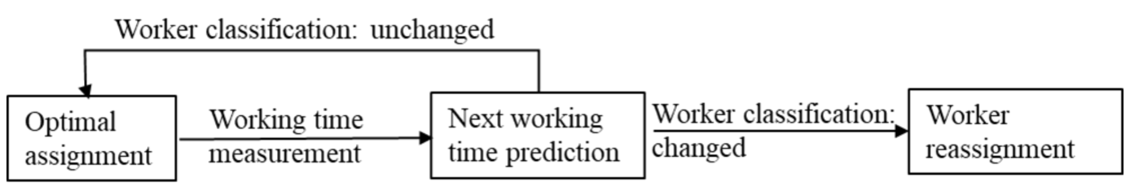


Figure 3.6: Description of optimal assignment of workers on staffing.

In the process of on-site management, timely adjustment of non-optimal assignments to make them optimal is of great help to improve production efficiency. Therefore, predicting the working time of workers in advance can help on-site managers to make timely and correct judgments on the necessity of redoing the optimal assignment of workers.

## 3.6 Summary

This chapter focus on the fact that workers have different processing capabilities for different processes in the reset limited-cycle model with multiple periods. Firstly, a method was proposed for calculating the total expected cost analytically when workers have different processing capabilities for different processes as a theorem. Secondly, a method was proposed for calculating the total expected cost where there are two types of processing capabilities (proficient or inefficient) of a worker for each process. And then, based on this assumption, theorems of the rule of the optimal worker assignment were proposed in the following two cases. In one of the cases, workers can be assigned to the tasks they are proficient at performing. The optimal assignment allows every worker to be assigned to the process they are proficient at performing. This theorem was proved mathematically. In one of the cases, an inefficient worker is assigned only to one task, but a proficient worker can be assigned to all the remaining tasks. The theoretical optimal assignment of workers is proved by a recursive method. It is the optimal assignment that inefficient workers should be placed either at the starting point or at the ending point of the process that can assign the inefficient workers, and proficient workers should be positioned at the remaining points of the process. Furthermore, the validity of the theorem was confirmed through numerical experiments. Finally, a method was proposed to reallocate workers according to the difference between the planned and measured values by measuring the working time of each process.

# Chapter 4 A work analysis for assessing workers' performance

## 4.1 Introduction

The work analysis system measures the working time necessary for assessing work performance. Video analysis requires a lot of time and labor to work analysis. In addition, when the same worker performs the same task, the variation in working time is reported to be about 10%[100]. Therefore, even if the analysis is performed precisely by video analysis, it is said that it is necessary to recognize that there is a variation of about 10% due to the difference in the measured timing.

In the line production of relatively small items such as home appliances, it is common to carry out continuous production and convey the parts on a conveyor to reduce travel time. On the other hand, in assembly lines such as automobiles, where there are many parts to be attached per process, even in continuous production, each process may have a moving section of several meters. In addition, in medium-mix, medium-volume production, batches can be produced by grouping similar items together, and manual inter-process transfer can be performed if the process steps between batches are not similar, or if the transport is infrequent. For tasks that do not involve moving, it is necessary to determine the start and end of the task only by hand movements. At present, based solely on the acceleration of the dominant hand, it is difficult to determine the start and end of a task. If the work involves movement, it is possible to observe a marked difference in the pattern of acceleration between walking and manual work. In this chapter, it is assumed that inter-process transportation between processes is done by walking.

Chapter 4 develops an automated work analysis system that makes use of an ultrasonic network, a tracking camera, and an acceleration sensor to access workers' performance. The objective is to propose a system that would be more than 90% consistent with the results of traditional video analysis. Position measurement and motion analysis are the two components of the work analysis approach. With the proposed method, an ultrasonic network and a tracking camera are used for measuring a worker’s position while the worker is moving, and an acceleration sensor is used for checking the acceleration of the dominant working hand. An application example is conducted to evaluate the viability of the suggested methodology. In this experiment, using the proposed work analysis system, it is possible to identify the start and end times of operations such as manual work, walk, loading/unloading, and stop. Furthermore, the assessment of workers' performance based on work analysis is discussed.

## 4.2 Proposed method

An automatic work analysis system using ultrasonic sensors, a tracking camera (hereafter, TC), and an acceleration sensor is proposed. The method uses feature amounts of a position on a flow line and an acceleration waveform of a wrist as explanatory variables for estimating a worker's movement in an assembly line.

Section 4.2.1 introduces the configuration of the work analysis system. Section 4.2.2 and Section 4.2.3 explain the method of measuring the flow line by using an ultrasonic sensor network and a TC. Liang et al.[101] proposed a method of measuring a position of a transmitter while successively selecting an ultrasonic receiver which is used for position measurement in a work environment where there are many objects such as shelves and pillars that prevent ultrasonic propagation. However, it is found that even the approach struggles to accomplish position measurement using just an ultrasonic sensor network in areas where ultrasonic signals are easily shielded. Thus, Section 4.2.2 explains the method of position measurement using an ultrasonic sensor network and formalizes the method. Section 4.2.3 proposes a process for connecting a flow line measured by the ultrasonic sensor and a flow line measured by the TC. Section 4.2.4 proposes a new process for selecting an estimation model. The process uses the feature amounts of the positions obtained from the flow lines and the acceleration waveform of the wrist as the explanatory variables for estimating the switching of the movements observed in the work.

### 4.2.1 The measurement system

Figure 4.1 shows the proposed system, which includes an ultrasonic sensor network, a TC, an acceleration sensor, a note personal computer (hereinafter, PC), and a small-sized personal computer (hereinafter, small PC). As shown in the figure, the ultrasonic network includes a transmitter attached to the worker’s helmet and several receivers installed on the walls of the indoor worksite. A TC is also attached to the worker’s helmet.

An acceleration sensor is carried on the wrist of the worker's dominant hand. The small PC included in the system is connected to the TC. It is used to supply power to the TC and to store the location information that is being measured. A PC is used to get data from the ultrasonic sensor network, the TC, and the acceleration sensor. The ultrasonic sensors and the small PC are connected to the PC via a wireless local-area network (LAN).

An ultrasonic sensor network made by Marvelmind was used in the system. Each ultrasonic sensor can transmit and receive ultrasonic waves over the entire hemispherical surface with a radius of 30 miles, and the distance between sensors can be measured at 7 Hz. The position of the transmitter establishes the position of the worker. The ultrasonic network can be used for positioning when the worker is in an area with few obstacles, where transmission of the ultrasonic signals will not be greatly affected. If the worker enters a non-line-of-sight area such as a narrow passage where a signal mask is very likely to occur, the TC, rather than the ultrasonic transmitter, is used for the dynamic positioning and tracking of the worker.

A RealSense Tracking Camera T265 made by Intel was used as the TC. The TC connected to the small PC is attached to the upper part of the helmet facing the side, and the position of workers is measured by the TC in environments where it is difficult to transmit and receive ultrasonic waves, such as narrow passages between shelves and behind columns.

An AccStick6 with a sampling frequency of 47.8 Hz and 15-gram weight made by SysCom was used as the acceleration sensor. The acceleration sensor measures acceleration in three axes. It has a built-in battery and flash memory and can be used alone during measurement. Measurement start and data transfer are performed using the attached adapter.

The PC executes the execution file for the ultrasonic sensor and the execution file for the TC by remote control of the small PC. By executing these simultaneously, both data are synchronized. The execution file for the ultrasonic sensor receives ultrasonic arrival time data from the ultrasonic transmitter to the receiver, calculates the position of the transmitter, and records the position data in the ROM of the PC. The execution file for the TC calculates the position of the TC and records the position data in the ROM of the small PC. In this system, each sensor operates independently, and after the measurement is completed, each position data is gotten from ROM by the execution file for motion analysis, and the process of a work analysis is started.

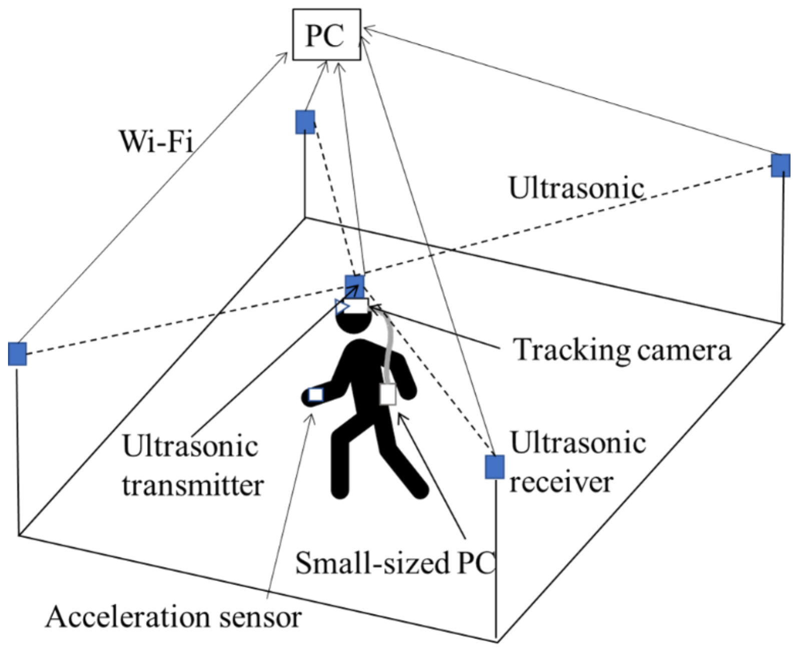


Figure 4.1: Configuration of the work analysis system.

### 4.2.2 Position measurement using an ultrasonic sensor network

The position of the ultrasonic transmitter can be calculated using the transmission time of the signal from the transmitter to the receiver[101]. The trigger signal is transmitted from a PC to all the ultrasonic transmitters and receivers in the system. Once the transmitter receives the trigger signal, it transmits pulse signals to its surroundings. For each receiver $p_k(k = 1, \dots, n)$, the time difference $t_k(k = 1, \dots, n)$ from the reception of the trigger signal to the arrival of the pulse signal is measured. If the speed of sound is assumed as $c$, then the distance $l_k$ between transmitter $p_0$ and receiver $p_k$ is $ct_k$. The positional relation of the transmitter and the receivers is represented in Figure 4.2. Here, $(x, y, z)$ is the position of transmitter $p_0$, and $(x_k, y_k, z_k)$ is the position of receiver $p_k$. The distance $l_k$ between the transmitter $p_0$ and the receiver $p_k$ can then be calculated using Eq. (4.2.1), a quadratic equation with the position $(x, y, z)$ as an unknown variable.

$$l_k^2 = (x_k - x)^2 + (y_k - y)^2 + (z_k - z)^2 \tag{4.2.1}$$

Using Eq. (4.2.1), $(l_k^2 - l_1^2)$ is then sequentially calculated for different values of $k$ $(k = 2, \dots, n,\ n \geq 4)$; Eq. (4.2.2) can be obtained by integrating and rearranging the terms.

$$\begin{pmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) & 2(z_2 - z_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) & 2(z_3 - z_1) \\ & \vdots & \\ 2(x_n - x_1) & 2(y_n - y_1) & 2(z_n - z_1) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_2^2 + y_2^2 + z_2^2 - l_2^2 - x_1^2 - y_1^2 - z_1^2 + l_1^2 \\ x_3^2 + y_3^2 + z_3^2 - l_3^2 - x_1^2 - y_1^2 - z_1^2 + l_1^2 \\ \vdots \\ x_n^2 + y_n^2 + z_n^2 - l_n^2 - x_1^2 - y_1^2 - z_1^2 + l_1^2 \end{pmatrix} \tag{4.2.2}$$

Eq. (4.2.2) is a simultaneous linear equation with unknown variables $(x, y, z)$. Based on the concept of the pseudo-inverse matrix, position $X$ of the transmitter $p_0$ is obtained by Eq. (4.2.3).

$$X = (A^T A)^{-1} A^T B \tag{4.2.3}$$

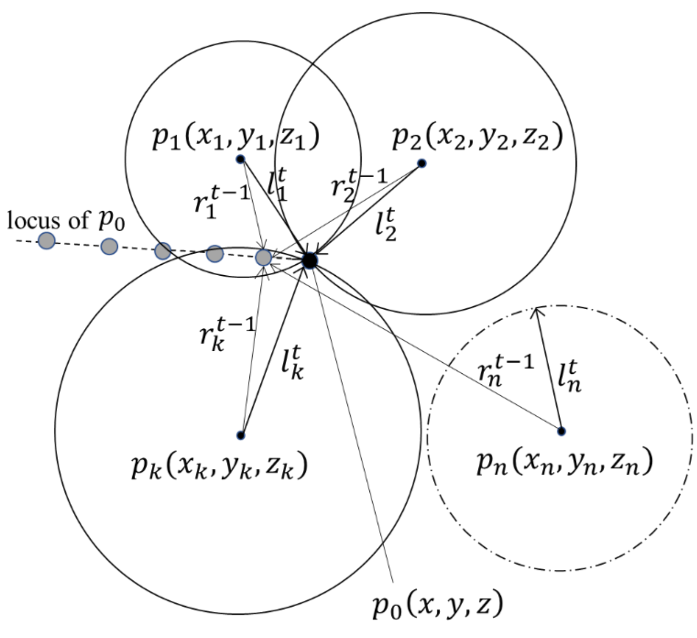

Figure 4.2: Positional relationship between transmitter and receivers.

The pseudo-inverse matrix is used for two reasons. The first reason is that since matrix $A$ is not a square matrix, the pseudo-inverse matrix is used to obtain a solution. The second reason is that the solutions obtained by using the pseudo-inverse matrix are consistent with those of linear simultaneous equations obtained by the least-square approach. In fact, some receivers incorrectly measure the distance to the transmitter. This method assumes that all circles intersect at one point, but the assumption does not hold in this case. Since the least-square approach can estimate a position having a minimum sum of squares of errors in the distance measured by each receiver, the pseudo-inverse matrix is used.

In reality, obstacles such as shelves and pillars hinder the propagation of ultrasonic signals inside an assembly plant. When a worker is in an area behind these obstacles, the arrival time of the ultrasonic signals will be inaccurate, and the estimated distance is likely to be different from the actual value. Therefore, the method described below is proposed to reduce the position estimation error.

Given the difficulty of judging whether a receiver is communicating correctly with the transmitter, the following proposed method offers a way to sequentially select receivers for position estimation. Figure 4.2 shows an illustrative example of how the position of transmitter $p_0$ can be estimated; here, four circles are drawn, each with a radius equal to the distance $l_k$ between the receiver $p_k$ to the transmitter $p_0$. As shown in the figure, the $t$-th estimated position $(x^t, y^t, z^t)$ of transmitter $p_0$ can be estimated as the intersection point of the multiple circles centered on the position of each receiver $p_k$, where the radius is the measured distance $l_k^t$ from receiver $p_k$ to transmitter $p_0$ at that time.

At the time of $t$-th data acquisition, the distance $l_k^t$ from the transmitter $p_0$ to the $k$-th receiver $p_k$ and the actual distance $r_k^{t-1}$ estimated from the $(t-1)$th measurement data are known. With regard to the receiver $p_k$ used for position estimation, an actual distance is given by $r_k^{t-1} = l_k^{t-1}$. On the other hand, with regard to the receiver which was not used for position estimation because of its measurement error, the distance from the position of the estimated transmitter to the receiver is obtained and regarded as the actual distance. The difference $d_k^t$ between the distance $l_k^t$ and the actual distance $r_k^{t-1}$ is calculated and used as the amount of change experienced by the transmitter $p_0$. The amount of change $d_k^t$ is given by the following equation.

$$d_k^t = |r_k^{t-1} - l_k^t| \tag{4.2.4}$$

Preliminary experiments confirmed that the amount of change $d_k^t$ was significantly increased when the measured value $l_k^t$ included a measurement error due to the reflection of ultrasonic waves on equipment in the measurement range. Therefore, the receiver with a small amount of change $d_k^t$ is preferentially used for position estimation.

According to the method, all the receivers are initially used for the estimation, and values of the obtained measurement errors are arranged from small to large, represented as $(d_{k1}^t, d_{k2}^t, \cdots, d_{kn}^t)$, where $d_{ku}^t < d_{ku+1}^t$, $u = 1,2,\cdots,n-1$. The three receivers with the smallest measurement error $(d_{k1}^t, d_{k2}^t, d_{k3}^t)$ are selected to estimate the $t$-th position, represented by $(x^t, y^t, z^t)$.

Eq. (4.2.2) assumes that there is no error in the measured distance from the ultrasonic transmitter to each of the receivers. Eq. (4.2.2) makes it clear that the solutions of three unknowns $(x, y, z)$ can be obtained if there are three or more linear equations. In other words, if there are three receivers capable of measuring a distance to the ultrasonic transmitter without making any error, three line segments that pass through the intersection of two circles in Figure 4.2 can be obtained, and thus, the solutions of the three unknowns can be obtained. The simultaneous linear equations in this case are expressed by Eq. (4.2.5).

$$\begin{pmatrix} 2(x_{k2}^t - x_{k1}^t) & 2(y_{k2}^t - y_{k1}^t) & 2(z_{k2}^t - z_{k1}^t) \\ 2(x_{k3}^t - x_{k1}^t) & 2(y_{k3}^t - y_{k1}^t) & 2(z_{k3}^t - z_{k1}^t) \\ 2(x_{k3}^t - x_{k2}^t) & 2(y_{k3}^t - y_{k2}^t) & 2(z_{k3}^t - z_{k2}^t) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (x_{k2}^t)^2 + (y_{k2}^t)^2 + (z_{k2}^t)^2 - (l_{k2}^t)^2 - (x_{k1}^t)^2 - (y_{k1}^t)^2 - (z_{k1}^t)^2 + (l_{k1}^t)^2 \\ (x_{k3}^t)^2 + (y_{k3}^t)^2 + (z_{k3}^t)^2 - (l_{k3}^t)^2 - (x_{k1}^t)^2 - (y_{k1}^t)^2 - (z_{k1}^t)^2 + (l_{k1}^t)^2 \\ (x_{k3}^t)^2 + (y_{k3}^t)^2 + (z_{k3}^t)^2 - (l_{k3}^t)^2 - (x_{k2}^t)^2 - (y_{k2}^t)^2 - (z_{k2}^t)^2 + (l_{k2}^t)^2 \end{pmatrix} \quad (4.2.5)$$

The method described above is intended for use in assembly plants where the transmission and reception of ultrasonic signals are only slightly affected by obstacles. However, this method cannot be used to get a worker's position if they move to an area where most of the receivers are unable to receive ultrasonic signals, such as behind a pillar or in an adjacent area. A TC-based approach to dynamic positioning is added to the system to address this issue.

### 4.2.3 Method of correction of location obtained by the tracking camera

For areas with no obstacles, the transmission and reception of ultrasonic waves are unimpeded, and the errors accumulated for position estimation will be minimal. Since the TC uses an image processing device for positioning, it has the advantage of being able to measure the position of the positioning target even in places where it is difficult to transmit and receive ultrasonic waves. In the proposed system, another important device, the TC, is used for measuring movement from a point where the worker leaves the area in which measurements can be made using ultrasonic sensors to a point where the worker returns. According to Intel, the TC has an error of about 1%. However, if even small errors accumulate, it becomes difficult to measure flow lines. Since the TC accumulates measurement errors in the position and movement direction that occur during operation, there is a problem that the flow line measurement error increases when using it for a long time. Therefore, a method of correcting the accumulation of measurement errors by the TC using the positioning data obtained by the ultrasonic sensor is described. Here, the movement line measured by the TC is labeled the TC line; the line measured by the ultrasonic sensors is labeled the US line.

First, a position (switching line) for a switchover from the ultrasonic sensor to the TC is set, and when the ultrasonic transmitter $p_0$ reaches the switching line, the traveling direction and the coordinates of the ultrasonic transmitter are made to coincide with those of the TC. Next, when the TC returns to the measurement area of the ultrasonic sensor network and reaches the switching line again, an angle of deviation $\theta$ between the traveling directions of the ultrasonic transmitter and the TC is calculated. The flow line of the TC is corrected according to the deviation angle $\theta$. The calculation method of $\theta$ is as follows.

(1) In the ultrasonic sensor measurement range, a line segment parallel to either the $x$-axis or the $y$-axis is set as the switching line. Figure 4.3 shows a schematic diagram when the switching line is set parallel to the $y$-axis. The black dots in the figure represent the coordinates of measurement points, and the broken line represents a flow line connecting the measurement points. Detect a corner closest to the switching line on the flow line measured by the ultrasonic sensor network and the flow line measured by the TC, respectively. Since the starting position of the TC is the origin, it is necessary to make the coordinates of the two flow lines coincide with each other somewhere on the flow lines. Therefore, obtain the coordinates of the corners of the two flow lines in front of the switching line so that their coordinates coincide with each other. After measuring the flow lines using the two sensors, detect their corners offline. The detection of a corner on the flow line measured by the ultrasonic sensor network is explained below. Let $p_u^0$ denote the position when the ultrasonic transmitter arrives $c$ (m) in front of the switching line, and find a measurement point closest

to the position away from that position by the Euclidean distance $c$, and let $p_u^{b2}$, $p_u^{b1}$ denote the two measurement points. Next, obtain an angle $\theta_u$ formed by the vectors $\overrightarrow{p_u^0 p_u^{b2}}$ and $\overrightarrow{p_u^0 p_u^{b1}}$ using the inner product. $\theta_u$ is expressed by Eq. (4.2.6).

$$\theta_u = cos^{-1}\left(\frac{\overrightarrow{p_u^0 p_u^{b2}} \cdot \overrightarrow{p_u^0 p_u^{b1}}}{\left|\overrightarrow{p_u^0 p_u^{b2}}\right| \cdot \left|\overrightarrow{p_u^0 p_u^{b1}}\right|}\right) \tag{4.2.6}$$

(2) $\theta_u$ has a value close to π in a section where the flow line is close to a straight line, and $\theta_u$ has a value close to $\pi/2$ in the corner. By tracing back to the flow line from in front of the switching line, find the position $p_u^*$ where $\theta_u$ is smallest, and determine that it is the corner. With regard to the flow line measured by the TC, use the same procedure to find the position of the corner closest to the switching line and let $p_t^*$ denote the position.

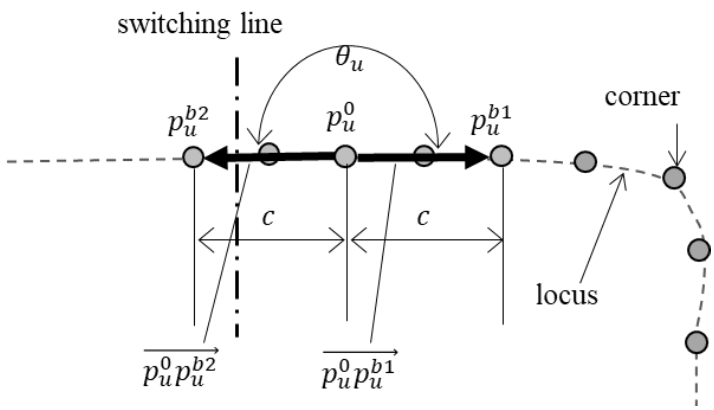


Figure 4.3: Corner detection near the switching line.

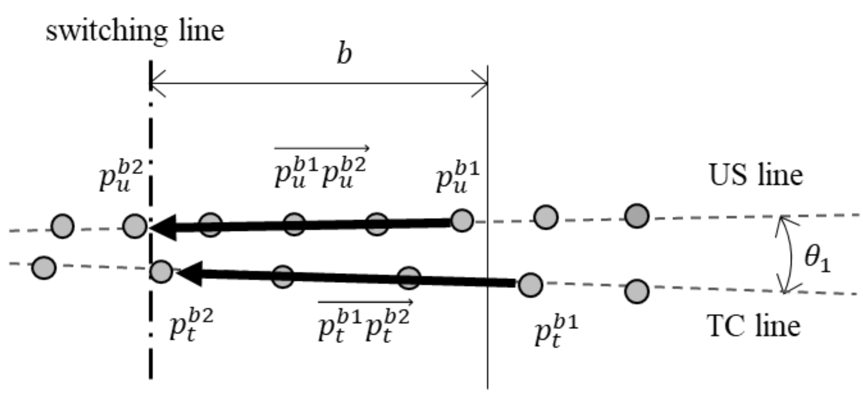


Figure 4.4: Positions and angles of inclination of the two flow lines on the switching line.

Find the difference $d_x$ in the $x$-axis direction and the difference $d_y$ in the $y$-axis direction between the corner positions $p_u^*$ and $p_t^*$. Then, by parallelly translating the entire flow line $(d_x, d_y)$ measured by the TC, make the coordinates of the corners closest to the switching line coincide with each other.

When the measurement points by the TC are expressed as $(x_t^i,\ y_t^i)$ $(i = 1, \ldots, NT)$, the coordinates $(x1_t^i, y1_t^i)$ of each of the measurement points after being translated is given by the following equation.

$$\begin{pmatrix} x1_t^i \\ y1_t^i \end{pmatrix} = \begin{pmatrix} x_t^i \\ y_t^i \end{pmatrix} + \begin{pmatrix} dx \\ dy \end{pmatrix}, \quad (i = 1, \ldots, NT) \qquad (4.2.7)$$

(3) Next, make the angles of inclination (traveling direction) of the flow lines measured by the two sensors coincide with each other on the switching line (Figure 4.4).

From among the measurement points by the ultrasonic sensor, find a measurement point $p_u^{b2}$ with the closest Euclidean distance from the switching line and a measurement point $p_u^{b1}$ closest to a position $b$ (m) in front of the switching line, and then obtain a vector $\overrightarrow{p_u^{b1} p_u^{b2}}$ that connects both ends. With regard to the measurement points by the TC, follow the same procedure to obtain $p_t^{b2}$ and $p_t^{b1}$ and then obtain a vector that connects both ends $\overrightarrow{p_t^{b1} p_t^{b2}}$. Obtain the angle $\theta_1$ formed by both vectors from the above Eq. (4.2.6).

(4) Next, rotate all the measurement points of the TC by $\theta_1$ around the origin of the coordinate system of TC. If the rotation matrix is $R$, the coordinates $(x2_t^i, y2_t^i)$ after the rotation of each of the measurement points $(x1_t^i, y1_t^i)$ $(i = 1, \ldots, NT)$ of the TC are given by the following equation.

$$\begin{pmatrix} x2_t^i \\ y2_t^i \end{pmatrix} = R \begin{pmatrix} x1_t^i \\ y1_t^i \end{pmatrix} = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ sin(\theta_1) & \cos(\theta_1) \end{pmatrix} \begin{pmatrix} x1_t^i \\ y1_t^i \end{pmatrix} \qquad (4.2.8)$$
$$(i = 1, \ldots, NT)$$

where the total number of measurement points in the measurement section by the TC is denoted by the symbol $NT$. Use Eq. (4.2.8) to make the angles of inclination of the two flow lines on the switching line coincide with each other. Figure 4.5 shows a schematic diagram when the angles of inclination of the two flow lines coincide with each other. At this time, the y-coordinates of the two flow lines on the switching line are misaligned.

Let $dy1$ denote this amount of misalignment, replace $dy$ and $dx$ in Eq. (4.2.7) with $dy1$ and 0, respectively, and parallelly translate the flow line by the TC again, and obtain the flow line $(x3_t^i, y3_t^i)$ after the parallel translation.

By the above procedure, the angles of inclination and the positions of the flow line measured by the ultrasonic sensor and the flow line measured by the TC are aligned with the switching line.

(5) Next, when the ultrasonic transmitter $p_0$ returns to the switching line again, obtain the angle of misalignment $\theta$ between the traveling direction measured by the ultrasonic sensor network and the traveling direction measured by the TC. Since this angle of misalignment $\theta$ is calculated offline, it is assumed that when the ultrasonic transmitter crosses the switching line into the measurement range of the ultrasonic sensor network, all the measurement points are known. The method described in (3) above is used.

From among the measurement points by the ultrasonic sensor, find a measurement point $p_u^{b2}$ closest to the switching line by the Euclidean distance and a measurement point $p_u^{b1}$ closest to a position $b$ (m) advanced from the switching line, and then obtain a vector that connects both ends $\overrightarrow{p_u^{b2}p_u^{b1}}$. With regard to the measurement points by the TC, use the same procedure to obtain $p_t^{b2}$ and $p_t^{b1}$ and then obtain a vector that connects both ends $\overrightarrow{p_t^{b2}p_t^{b1}}$. Obtain the angle $\theta$ formed by both vectors from the above Eq. (4.2.6).

Use the above procedure to obtain the angle of misalignment $\theta$ between the traveling directions of the two flow lines when the ultrasonic transmitter $p_0$ returns to the switching line again.

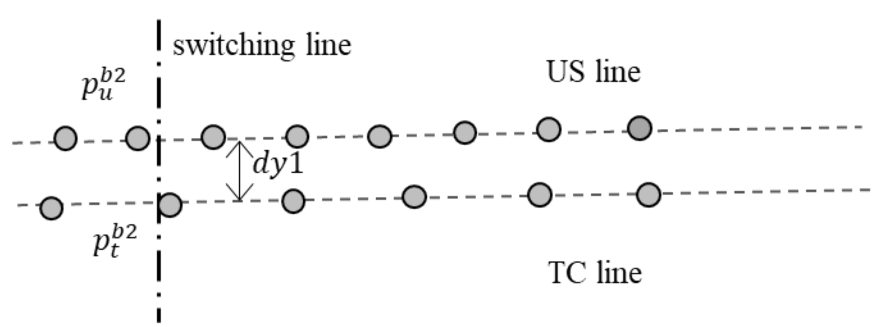


Figure 4.5: Misalignment of two flow lines on the switching line.

A schematic diagram of the correction method is shown in Figure 4.6. The boundary line (the vertical dash-dot-dash line in Figure 4.6) for discriminating whether the ultrasonic sensors should be used for positioning is preset using a preliminary experiment. When the worker moves from the area where the TC is used and enters the measurement range where the ultrasonic sensors are used, it is necessary to correct the movement line to ensure that it is consistent with the actual route.

When the worker arrives at the boundary, the TC line is rotated so that the traveling direction of the US line and the TC line is the same. A variable $\theta(rad)$ is defined as the difference in the moving direction between the US line and the TC line at the point where the worker returns from the area where the TC is used for measurement. During this period, if the number of positions recorded by the TC is $m$, the moving direction deviation for each position is assumed to be $\theta/m(rad)$. Under this assumption, each position data point is multiplied by the rotation matrix $R$ to correct the position coordinates. The rotation matrix is given by Eq. (4.2.9).

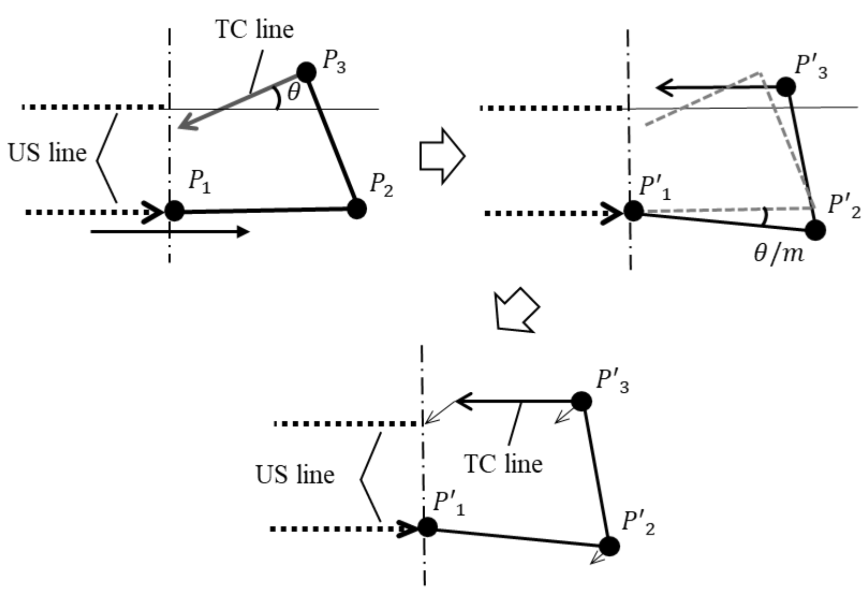


Figure 4.6: How to correct the movement line.

$$R = \begin{bmatrix} cos(\theta/m) & -sin(\theta/m) \\ sin(\theta/m) & cos(\theta/m) \end{bmatrix} \tag{4.2.9}$$

The coordinates for all the position data $(x_i, y_i), (i = 1, \cdots, m)$ recorded by the TC are represented in matrix $P$; the corrected coordinates $(X_i, Y_i)$ appear in matrix $P'$.

$$P = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_m & y_m \end{bmatrix}$$

$$P' = \begin{bmatrix} X_1 & Y_1 \\ X_2 & Y_2 \\ \vdots & \vdots \\ X_m & Y_m \end{bmatrix}$$

The position of the original point shown in the first row is the same in both matrices; that is to say, $P_1 = P_1'$. The position coordinates of the second and subsequent lines $(i = 2, \cdots, m)$ are corrected by Eq. (4.2.10).

$$P_i' = P_{i-1}' + (P_i - P_{i-1}) \cdot R \tag{4.2.10}$$

Having corrected the direction of the movement line using the above procedure, the next step is to correct the positional deviation.

In Eq. (4.2.11), $D$ represents the deviation between the position measured by the ultrasonic sensors, $(x_{us}, y_{us})$, and the position corrected by Eq. (4.2.10), $(X_m, Y_m)$, at the boundary line.

$$D = [dx, dy] = [X_m - x_{us}, Y_m - y_{us}] \tag{4.2.11}$$

The accumulated position estimation error obtained by the TC is divided by the number of data points $m$, and all the estimation data values are corrected proportionally, as shown in Eq. (4.2.12).

$$P' = \begin{bmatrix} X_1 - dx/m & Y_1 - dy/m \\ X_2 - 2dx/m & Y_2 - 2dy/m \\ \vdots & \vdots \\ X_m - dx & Y_m - dy \end{bmatrix} \tag{4.2.12}$$

Using the above procedure, when the worker returns from the area in which the TC is used to the area where the ultrasonic sensors are used, the movement line measured by the TC will be corrected so that the position measured by both methods is consistent.

### 4.2.4 Motion estimation

This section describes the method for estimating the motion by taking the work in an assembly plant that manufactures belt conveyors as an example. The item produced on this production line is a belt conveyor. Generally, inter-process transportation on an assembly line can be broadly divided into the use of belt conveyors and manual transportation. Since the product itself is small and can be carried by one worker, inter-process transportation between processes is done by walking at the assembly plant that manufactures belt conveyors. The operations involved in processing belt conveyors can be roughly divided into the following activities: stopping to confirm instructions, walking to the parts store, loading the parts, walking with parts to the workspace, unloading parts at the workspace, and assembling the parts by manual. The above activities are simplified into four actions, which were identified as (1) manual work, (2) walk, (3) loading/unloading, and (4) stop. Using a preliminary experiment, it was observed that the walking speed and the dominant hand movement (acceleration) are different for the four basic motions involved in the process. These four motions can be estimated using three independent variables based on position, moving speed, and acceleration data.

The acceleration of the dominant working hand for the different motions was obtained through a preliminary experiment, the results of which are shown as the fluctuating line at the bottom of Figure 4.7. Figure 4.7 is accompanied by a photo of the assembly plant producing the belt conveyor and a simulated experiment in a laboratory, corresponding to the above four motions. The solid line at the top of Figure 4.7 shows the transition of the four motions obtained by video analysis. The acceleration data at a point in time is produced by combining the acceleration of the 3-axis components, as recorded by an accelerometer worn on the wrist of the dominant working hand. The stop operation has a significantly smaller acceleration amplitude than the other operations. The amplitude of the walk operation is larger than that of the stop operation and smaller than that of the manual work operation. Both the manual work operation and the loading/unloading operation have large amplitudes, but the frequency is higher during the manual work operation. It is obvious from the graph that the amplitude and frequency are significantly different for the four motions. Generally, the short-time Fourier transform or wavelet transform can be used to analyze vibration waveforms with non-stationary frequencies; however, the wavelet transform has the advantage of supporting the analysis of vibration waveforms in a wider frequency band compared to the short-time Fourier transform[102]. Consequently, the wavelet transform was used here to obtain the acceleration features (continuous wavelet coefficients, or CWT coefficients) for each motion. At any point during the video analysis, it is important to develop an estimation model for discriminating the different motions using the obtained acceleration features, the position data, and the moving speed as independent variables.

If the number of independent variables representing the different features is $N$, then the input data can be represented by an $N$-dimensional vector $\vec{x} = (x_1, x_2, \ldots, x_N)^T \in R^N$. The dependent variable (output value) is defined as $y \in R$, which represents the motion category obtained from video analysis. Denoting the total number of input/output pairs as $M$ and the $i$-th input/output value pair as $(\vec{x_i}, y_i)$, the task is to find the best combination of the control variable $\vec{\omega}$ and the estimation model variable $\Theta$ according to the estimation accuracy using the input and output pairs.

$$\hat{y} = f(\vec{x}, \vec{\omega}, \Theta) \tag{4.2.13}$$

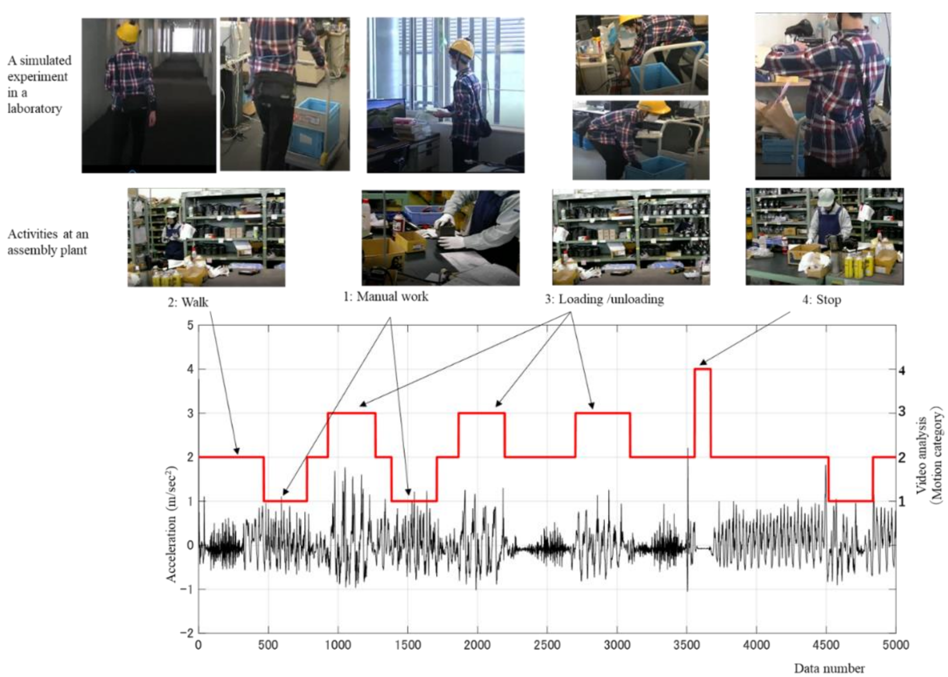


Figure 4.7: Example of acceleration of the dominant hand in an assembly plant that manufactures belt conveyors.

The total collection of input/output pairs is represented by data set $D = \{(\overrightarrow{x_1}, y_1), \dots, (\overrightarrow{x_M}, y_M)\}$. Data set $D$ is divided equally into $K$ groups. If the $u$-th testing data set is denoted as $C_u$ $(u = 1, \cdots, K)$, the $u$-th training data set $L_u$ can be obtained by removing testing data set $C_u$ from data set $D$, which can be expressed as $L_u = D \setminus C_u$. The training data set is used to determine the control variable of the estimation model. Using the determined estimation model, the input vector of the testing data set is then used to estimate the output value and the estimation accuracy. The measurement accuracy $A(\Theta)$ obtained by the $k$-fold cross-validation method is expressed as follows:

$$A(\Theta) = \sum_{u=1}^{K} \sum_{(\vec{x},y)\in C_u} I(y, \hat{y}) \,/M \qquad (4.2.14)$$

Here,

$$AI(y, \hat{y}) = \begin{cases} 1 \; if \; y = \hat{y} \\ 0 \; if \; y \neq \hat{y} \end{cases}$$

Based on the previous procedures, including dividing the data set $D$ into a training data set and a testing data set for different combinations of the data set, and finding the most fittable control variable $\vec{\omega}_u^*$ using all the training data sets $L_u = D \setminus C_u$, the estimation function can be expressed as $f(\vec{x}, \vec{\omega}_u^*, \Theta)$. Each testing data set having no relationship with the corresponding training data set is then used for estimation by the model; if the result is correct, the point is added to the value of $A(\Theta)$. In this way, the ability of the predictive model is asymptotically evaluated. The model with the largest $A(\Theta)$ value is selected as the estimation model by which the motion can be estimated from the newly input data set.

The method described above can be used to construct an automated work analysis system. The system for implementing the proposed method, including ultrasonic sensors, a TC and an accelerometer, is shown in Figure 4.1.

## 4.3 Application example

The working processing of an assembly plant that manufactures belt conveyors was simulated to test the measurement accuracy of the motion analysis using the proposed method. The method of motion analysis proposed in Section 4.2.4 is to discriminate the different motions using the obtained acceleration features, and position measurement data (i.e., position and moving speed) as independent variables. Therefore, it is first to know generally the measurement accuracy of position measurement. In the application example, the positioning accuracy of the method was first measured, and then the accuracy of the motion analysis was evaluated. Section 4.3.1 provides information on the experiment condition. Through preliminary experiments, the measurement accuracy of the proposed method was investigated from ultrasonic sensors and TC in Section 4.3.2. The measurement accuracy of motion analysis was described in Section 4.3.3.

### 4.3.1 Experiment conditions

Experiments were carried out in a rectangular room (8 m×9 m） and a long aisle (2.5 m×40 m) in a laboratory (Figure 4.8). In the rectangular room, ultrasonic sensors were used for dynamic tracking. Five ultrasonic receivers were installed in the positions marked with ○. In the aisle area, since the wall between the aisle and the rectangular room made the transmission and reception of ultrasonic signals difficult, a TC was used for positioning.

Figure 4.8 shows the test environment for the evaluation of the accuracy of measurement. The origin of the world coordinate system was set in the room. The origin of the coordinate system of the ultrasonic sensor was set to coincide with the world coordinate system. The coordinate system of the TC is constrained so that a position when the start command is executed is the origin. The switching line was set at the position of x= -3.0m. Regarding the arrangement of receivers in the diagram, the following points were taken into consideration. The receivers were arranged so that the worker could see three or more receivers when he looked inside from the entrance of the room. Due to the restriction that the entrance door opens into the room, receiver 1 was unable to be brought closer to the entrance door than it is now. In addition, since there was a partitioning screen in the room, receiver 3 was placed on the surface of the partitioning screen on the entrance side. The receivers were placed so that the area from the partitioning screen to the corridor side could be measured mainly by receivers 1, 2, and 3, and the area on the opposite side of the partitioning screen could be measured mainly by receivers 2, 4, and 5. Since receiver 2 was intended to be used in both of the areas, the layout shown in Figure 4.8 was chosen.

### 4.3.2 Position measurement

A male worker at the age of 23 participated in the preliminary experiment of position measurement. Five measurement points were set in the room and four in the corridor, and marks were put on the floor. The position of each measurement point was measured using a tape measure. The sampling rate of the ultrasonic sensor was 26Hz (its rated frequency) and that of the TC was 10Hz (its rated frequency). The movement path of the worker consists of the measurement points A-B-A-C-D-C-E-F-G-H-I-E-A in Figure 4.8, and the worker came to a standstill for 10 seconds at each measurement point. The movement path was a total of 114.6 meters, of which the range measured with TC was 85.2 meters, and the rest is measured by ultrasonic sensors.

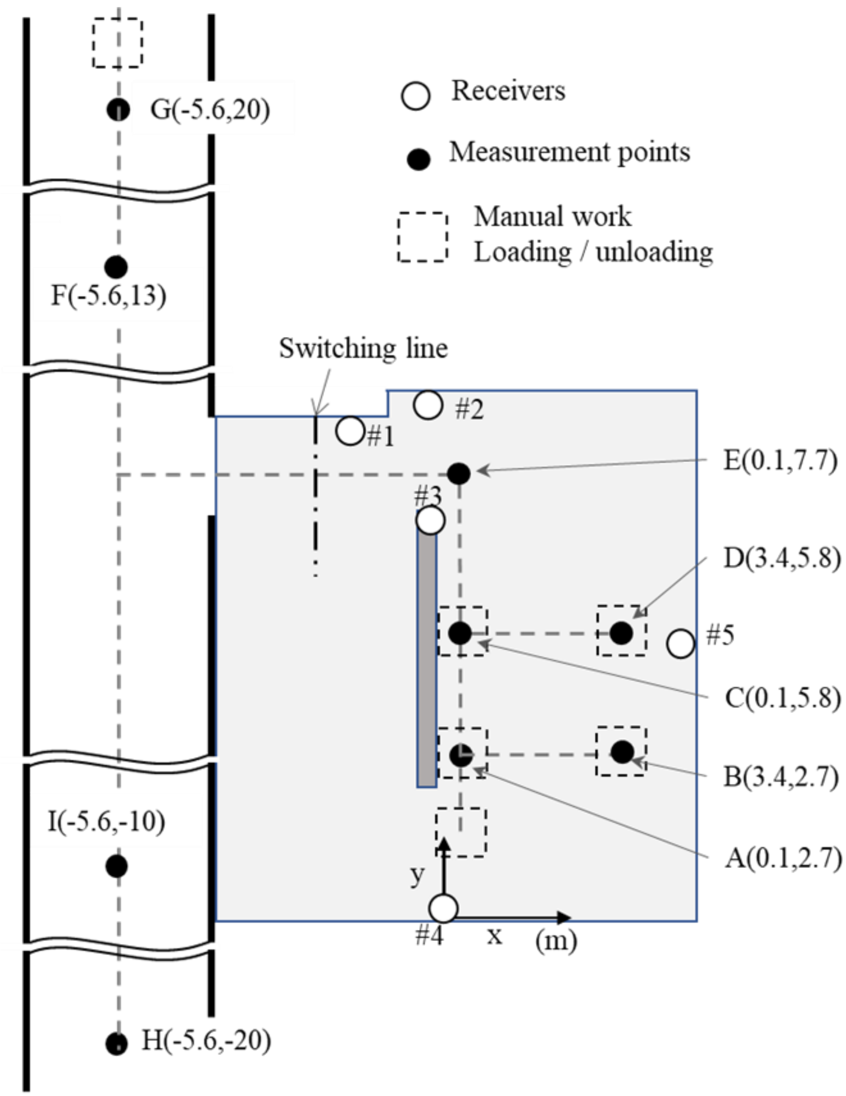


Figure 4.8: Test environment and measurement points.

Figure 4.9 shows a flow line measured by the ultrasonic sensor and a flow line measured by the TC. Since, with regard to the TC in Figure 4.9, the time when the start command is executed is the origin, the origin of the TC coincides with the origin of the world coordinate system. For the purpose of compensating for the deviation of the origins and a measurement error of the TC near the switching line, the above Eq. (4.2.6) is used to obtain corners closest to the switching line ($x$ = -3m) and each corner is marked by a circle. In addition, Figure 4.10 shows the relationship between the data numbers on the flow lines in the process of detecting the corners and the angle $\theta_u$ obtained by Eq. (4.2.6).

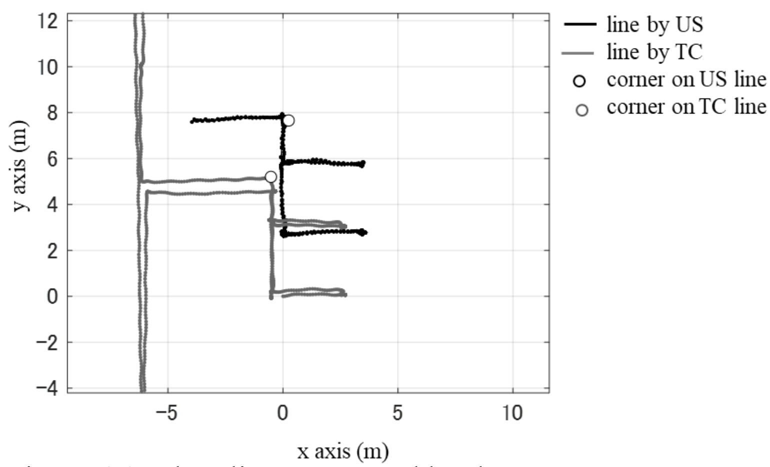


Figure 4.9: Flow lines measured by the two sensors.

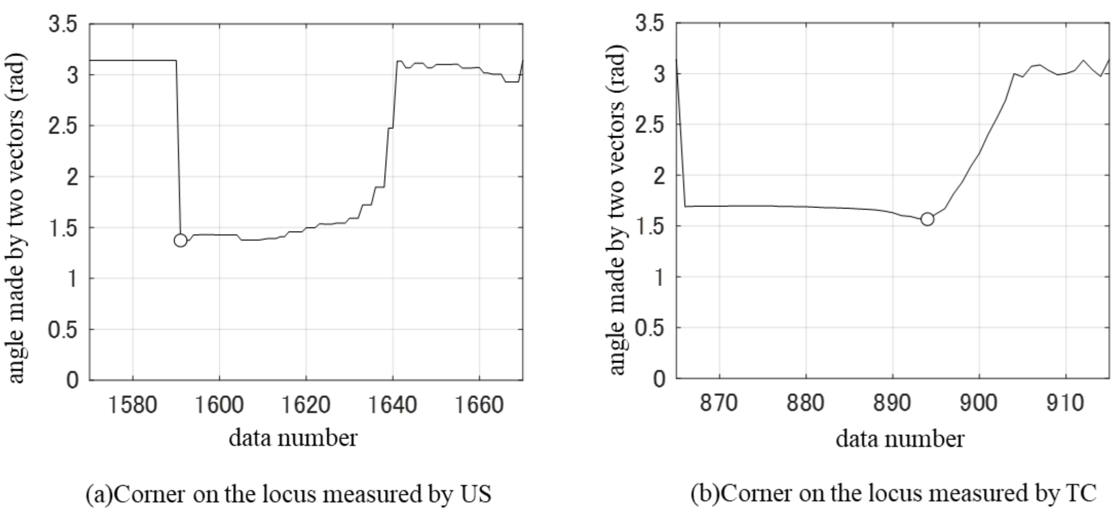


Figure 4.10: Location of corners on two flow lines.

Next, the difference between the coordinates of the corners of the flow lines was calculated. Since the difference $d_x$ in the $x$-axis direction and the difference $d_y$ in the $y$-axis direction was 0.77m and 2.45m, respectively, the entire flow line measured by the TC was parallelly translated by $(d_x, d_y)$ based on Eq. (4.2.7). The results are shown in Figure 4.11.

Next, steps (3) and (4) were performed to make the coordinates and the angles of inclination of the flow lines during the switchover from the ultrasonic sensor to the TC coincide with each other. The results are shown in Figure 4.12.

In this process, when the ultrasonic transmitter $p_0$ returned to the switching line again, the angle of misalignment $\theta$ between the traveling direction measured by the ultrasonic sensor and the traveling direction measured by the TC was -0.6 degrees (-0.001 rad), and the number of data $m$ measured from the switchover to the TC till the switchover to the ultrasonic sensor again was 1,161. The obtained $\theta$ and $m$ are brought into Eq. (4.2.9) of this thesis to correct the coordinate data measured by the TC. In Figure 4.12, it can be seen that the coordinates and the angle of inclination of the flow line measured by the ultrasonic sensor coincide with those of the flow line measured by the TC on the switching line ($x$= -3m).

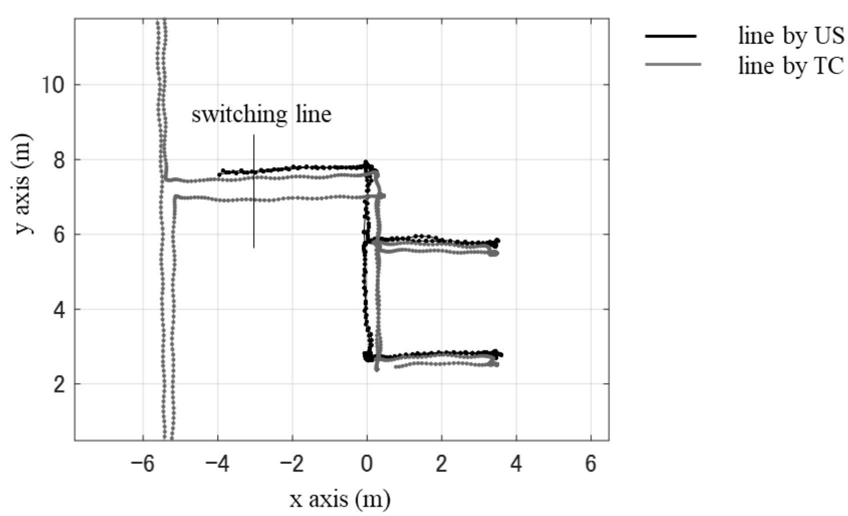


Figure 4.11: Result of the alignment of the two flow lines which start at the corners.

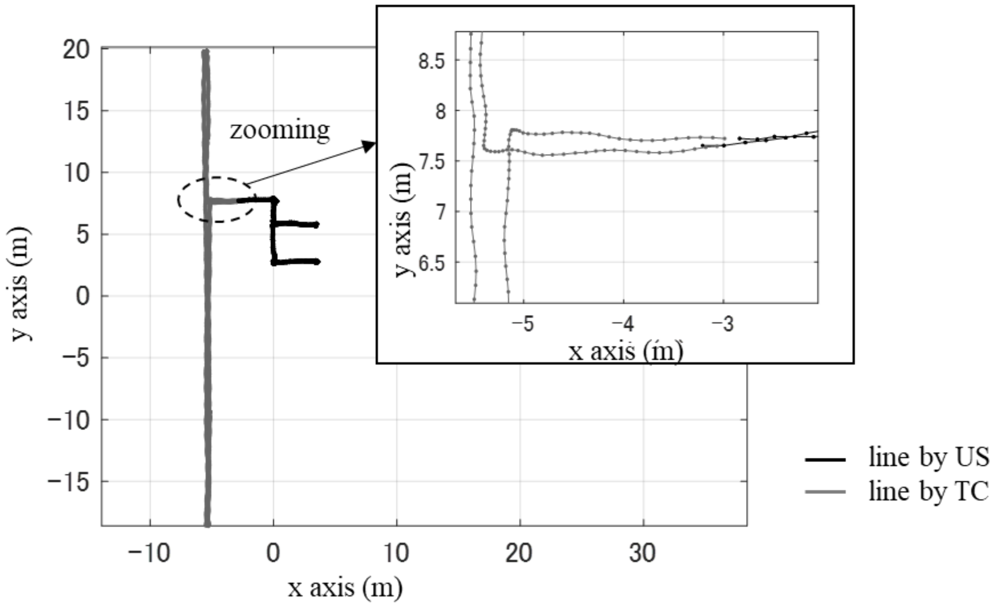


Figure 4.12: Result of making the positions and the angles of inclination of the two flow lines coincide with each other on the switching line.

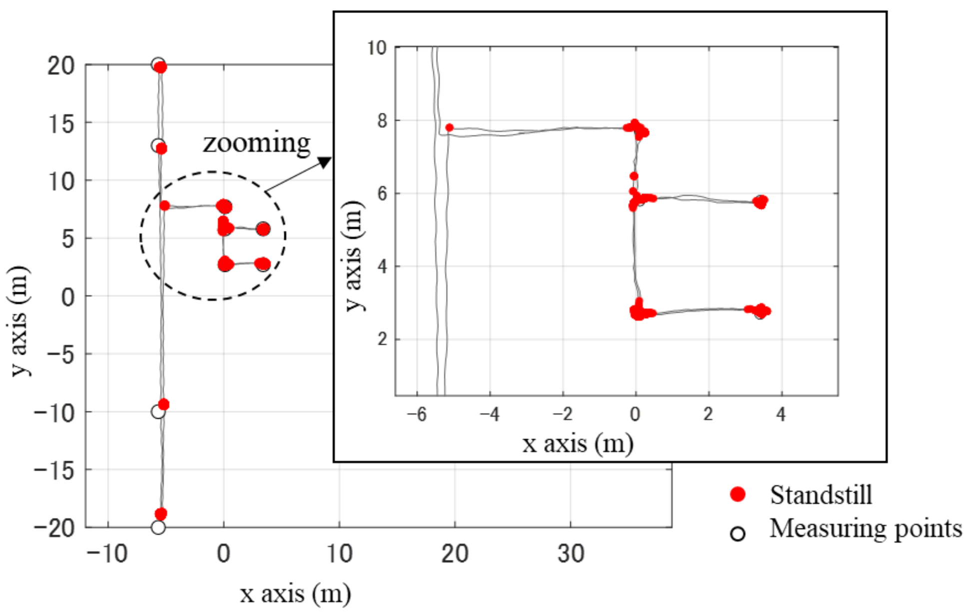


Figure 4.13: Unified flow line and stop positions.

Next, position measurement errors will be explained. When the moving speed was equal to or less than 2 cm/ sampling time interval on the synthesized flow line shown in Figure 4.12, the worker was determined to come to a standstill. The positions at which the worker was determined to come to a standstill are shown by gray dots in Figure 4.13.

Most of the standstill positions were near the measurement points, but when the worker changed his travelling direction by 90 degrees near the corridor (-5m, 7.7m) outside the room, he was sometimes determined to be at a standstill. Since the positions just outside the room were not used as measurement points, they were excluded from this analysis of measurement errors.

Next, with regard to the five measurement points set in the room, the Euclidean distances to standstill positions in the $x$-axis and $y$-axis directions were obtained and regarded as measurement errors of the ultrasonic sensor. The left diagram of Figure 4.14 shows a scatter plot of the measurement errors. In the same way, with regard to each of the four measurement points set in the corridor, the Euclidean distances to the standstill positions were calculated and regarded as the measurement differences of the TC. The right diagram of Figure 4.14 shows a scatter plot.

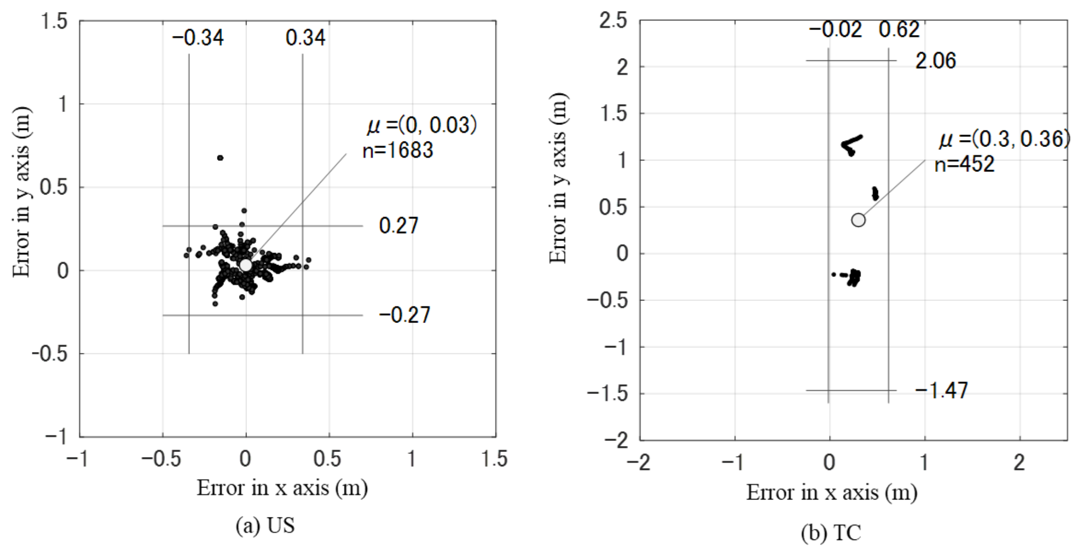


Figure 4.14: Position measurement errors by the US and the TC.

Table 4.1: Measurement time for each participant.

| Worker \ Number of experiments | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Participant A | 15 minutes | 15 minutes | 3 minutes | 3 minutes |
| Participant B | 15 minutes | 15 minutes | | |
| Participant C | 15 minutes | | | |

The average measurement error of the ultrasonic sensor was 0.0 m in the $x$-axis direction and its $3\sigma$ range was $\pm0.34$m. The average error in the $y$-axis direction was 0.0 m and its $3\sigma$ range was $\pm0.27$ m. In the MTM method used to set the standard time, the length of one step is regarded as 0.7 m. It was confirmed that the measurement errors of the ultrasonic sensor could be expected to be within one step. The average measurement error of the TC was 0.3 m in the $x$-axis direction and its $3\sigma$ range was $\pm0.3$m. The average error in the $y$-axis direction was 0.36 m and its $3\sigma$ range was $\pm1.7$ m. The location of Tc was corrected by the ultrasonic sensors, so the reason for the measurement error of TC often also covered the part of ultrasonic sensors. It was confirmed that the measurement errors of the TC could be expected to be within two steps when the worker moves 80m.

### 4.3.3 Motion analysis

Three male workers between the ages of 22 to 23 participated in the experiment on motion analysis. The three participants in the experiment were designated as participant A, participant B, and participant C, respectively. As shown in Table 4.1, participant A performed the operation four times, with respective times of 15 minutes, 15 minutes, 3 minutes, and 3 minutes. The positions where the motions of manual work and loading or unloading products occurred are marked by the □ symbol in Figure 4.8. Participant B executed the process twice, with a time of 15 minutes each time, and participant C performed the 15-minute operation once.

The independent variables used for motion analysis include position (values of the $x$-coordinate and $y$-coordinate), moving speed, and the CWT coefficient obtained by continuous wavelet analysis of the acceleration fluctuations. According to Olhede et al.[103], Morse wavelets are more effective for original wavelets with non-stationary signals. Therefore, Morse wavelets were used in the analysis of the acceleration data. The CWT command of MATLAB Wavelet ToolBox developed by MathWorks was used for wavelet analysis, and the data were processed offline. The actual sampling rate confirmed by the measurement data was 47.8 Hz. The maximum frequency that can be analyzed when performing wavelet analysis is 20.7511 Hz and the minimum frequency is 0.0032 Hz. The CWT function in MATLAB sets the scaling factor to divide this frequency interval into 127 steps by default, so the analysis was performed without changing any settings. In actual action, it is unlikely that the hand extension motion takes more than 10 seconds. Therefore, the CWT coefficients obtained from frequencies ranging from 0.1 to 20 Hz were used as the acceleration features for motion estimation. The number of features was 99. The position (x, y) and moving speed were combined into a data set consisting of 102 explanatory variables. Finally, 103 elements (102 explanatory variables,1 element of action recorded by a camera) were included in the input vector of each data set. In addition, after the data set has been acquired, the correlation coefficients among the explanatory variables were examined before creating the behavior estimation model. To prevent the multicollinearity of the variables, one of the explanatory variables with a correlation coefficient greater than 0.8 was removed. The actual number of explanatory variables was 48. Data for the position, moving speed and acceleration features were obtained during the seven iterations of the experiment; the data set for each time was obtained by associating these data with the motion determined by video analysis. Taking the data set of the first operation performed by participant A as the newly input data set, the other six data sets could then be used as training data sets. 23,403 data pairs were used in the training data sets, and 5,669 data pairs were used for the newly input data set.

During the experiment, the movements of the workers were recorded by a camera for motion analysis. In this experiment, the four basic motions of manual work, walk, loading/unloading, and stop were interpreted as follows: The manual work operation was the motion that brings items together. The loading operation was the motion of putting items into the folded container on the trolley, whereas the unloading operation was the motion of taking the item out of the folded container on the trolley. They were collectively referred to as loading/unloading. The walk operation included walking, walking with items, and walking while pushing a trolley. The stop operation was the motion that stops. Figure 4.15 shows the percentage of operating time for each of the four motions in the training data. The manual work operation, walk operation, loading/unloading, and stop operation were respectively 14%, 55%, 25%, and 6% in the training data.

Next, the estimation model was determined using the training data sets. The estimation accuracy of the linear discriminant function, decision tree, k-nearest neighbors (KNN), bootstrap aggregating tree, and subspace KNN for unknown data was tested by the cross-validation method. When the division number of data set K was set to 5, the validation accuracy of these classification models is shown in Table 4.2. The bootstrap aggregating tree model had the maximum estimation accuracy of 97.7% and was thus selected as the estimation model in this case.

As is shown in Table 4.2, the influence of the position coordinates on the accuracy of the model was considered in the selection of the estimation model. If the position coordinates were excluded from the explanatory variables, the estimation accuracy decreased slightly. From the viewpoint of estimation accuracy, the estimation accuracy was improved by including position information in the estimation model. Also, from the viewpoint of work analysis, it was possible to confirm whether or not each work was performed in a preset place if there is position information, so it thought that position information was necessary.

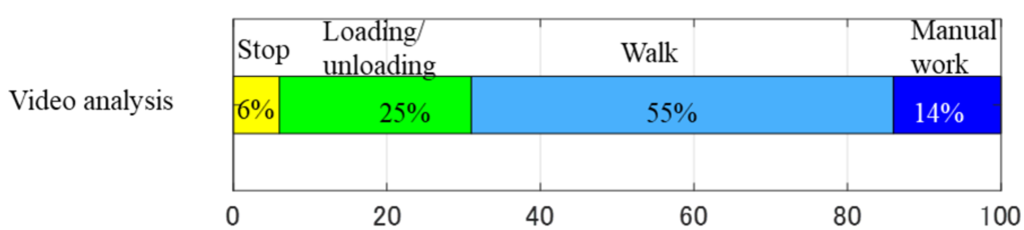


Figure 4.15: Percentage of operating time in the training data sets (three participants, 66 minutes, 23,403 data pairs).

Table 4.2: Accuracy of classification models (5-fold cross-validation).

| Classification models | Validation accuracy (%) (Position coordinates were not excluded from the explanatory variables) | Validation accuracy (%) (Position coordinates were excluded from the explanatory variables) |
|---|---|---|
| Bootstrap aggregating tree | 97.7 | 96.0 |
| K-nearest neighbors (KNN) | 96.9 | 96.7 |
| Subspace KNN | 96.7 | 97.0 |
| Decision tree | 92.9 | 86.2 |
| Linear discriminant function | 87.7 | 84.5 |

Using the selected model, the motion at different times was estimated for the newly input data set. Figure 4.16 shows the changes in acceleration of the dominant working hand and the wavelet analysis of the acceleration data with respect to the newly input data set (range of 0.1 Hz to 20 Hz used as features). The acceleration data are shown in the upper panel of the figure; the scalogram is shown in the lower panel. In each panel, the horizontal axis shows time, while the vertical axis shows the logarithm of the frequency. The higher the brightness, the larger the absolute value of the CWT coefficient.

The results of motion estimation using the chosen model are shown in the lower panel of Figure 4.17. For comparison, the results obtained by video analysis for the same activities appear in the upper panel. As can be seen here, the estimation error for the various motions was 1% for stop, 3% for loading/unloading, 4% for walk, and 0% for manual work. The time intervals for each motion are shown on the timeline in Figure 4.18. In all, 5,669 data pairs were used for the newly input data set. The concordance rate between the results obtained by the proposed estimation method and the results obtained by video analysis was 92.8%. Although video analysis is commonly used for work analysis, previous studies have attached no clear objectives to the accuracy of the estimates produced from such analysis. In this study, referring to the report by Fukuta et al. [100], the targeted estimation accuracy was defined as greater than 90% consistency with the results obtained by video analysis. Based on the estimation results displayed in Figure 4.17, the targeted accuracy level was successfully achieved.

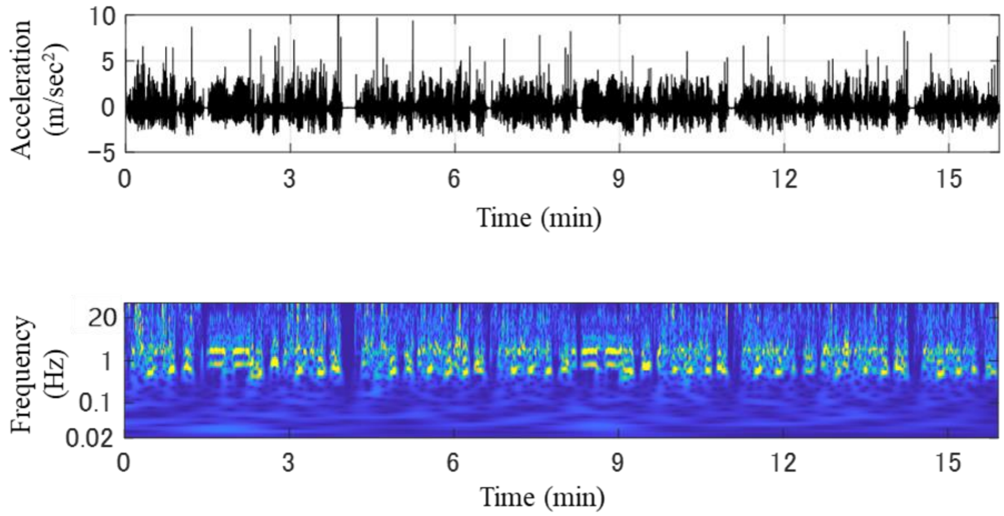


Figure 4.16: Results of acceleration and wavelet analysis for dominant hand in test data set.

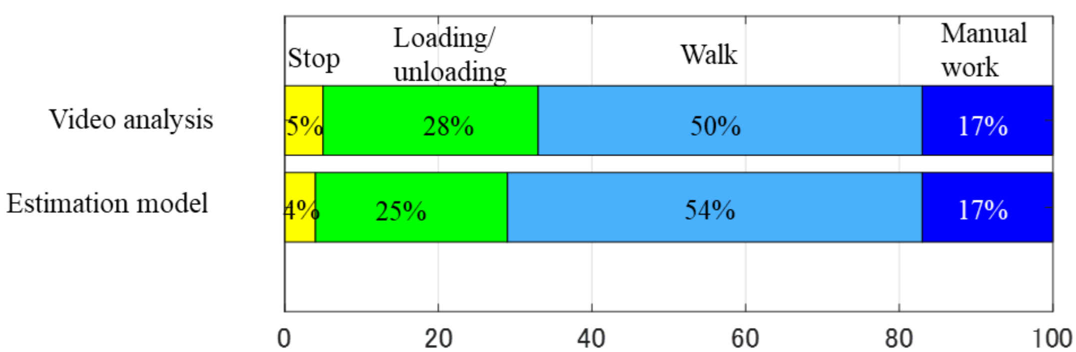


Figure 4.17: Comparison of percentage of time for each motion (one participant, 15 minutes, 5,669 data pairs).

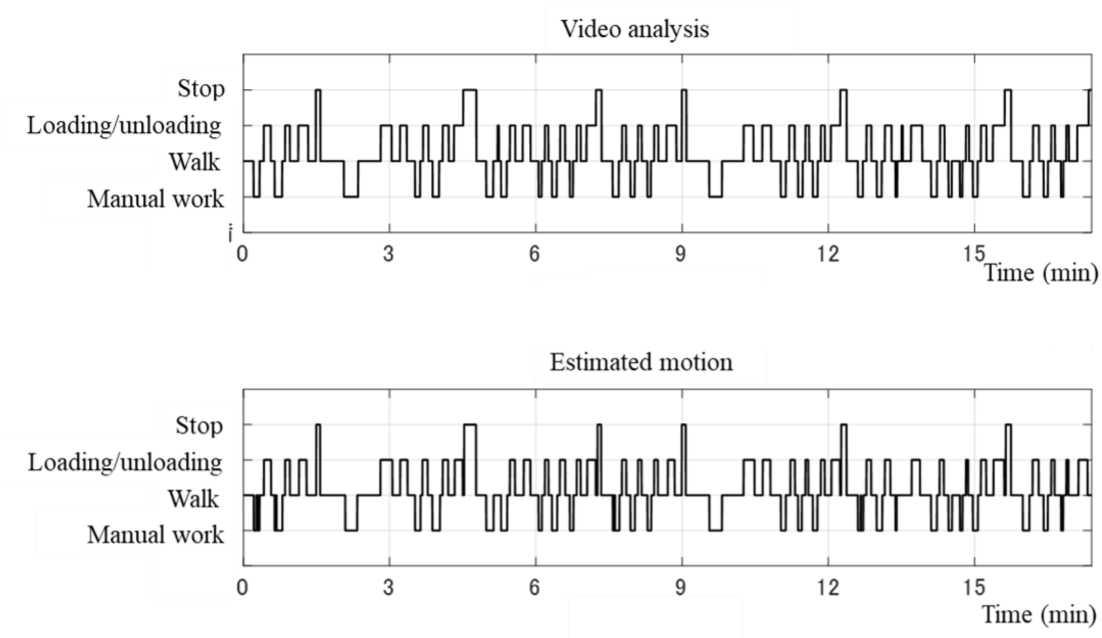


Figure 4.18: Estimation results for the test data set (changes in motion over time).

In the application example of Section 4.3.3, motion analysis is conducted in two phases. The first phase is the selection of an estimation model for motion estimation. The processing time for this first phase was about 60 seconds in this experiment. The second phase uses the selected estimation model to estimate the action on the test data set. In this stage, the processing time required for analysis, including graph display, was 10.7 seconds. It is considered to be within the range that will not be a major obstacle to continuous work analysis. An Intel (R) Core i7-9700 PC is used for calculations.

In addition, not only the motion but also the working time for each action can be measured by the proposed work analysis method. In the application example of Section 4.3.3, based on the estimation results for the test data, the manual work operation was detected 27 times, with an average working time of 5.79 seconds and a standard deviation of 2.67 seconds. The loading/unloading operation occurred 35 times, and the average work time per time was 6.34 seconds. The standard deviation was 2.60 seconds.

## 4.4 Assessment of workers' performance based on work analysis

Production workers have a direct impact on the efficiency, cost, and quality of production. Because his position is at the site of product manufacturing, it directly determines whether the production goals are met or not. Scheduling of production tasks and personnel ensures that production tasks are completed according to production delivery dates. The processing capability of the workers is different. On the same assembly line, even different workers in the same process will have different working times and quality. Although there are standard operations on the site of the assembly line, the capability of workers to perform is also affected by proficiency, personal physical conditions, and the surrounding environment, and there is a gap with standard operations.

This work analysis system can measure workers' working time and thus obtain a basis for assessing the workers' performance. First, the working standard is formulated. Then, by using this system, working time is measured, the worker's motions in a particular task are analyzed, and the actual work performance of the worker is assessed according to the performance standards that worker needs to achieve. Finally, feedback is provided to the worker. For workers with substandard performance, strengthen training, improve their unreasonable parts, eliminate performance defects, and standardize work. For workers who have accomplished well, encourage them to continue to maintain an excellent level of performance. The performance assessment enables to obtain of relevant and timely assessment information. The assessment information allows managers to keep abreast of how workers are carrying out their work and to grasp the effectiveness of the operation so that they can provide the necessary guidance to workers. It is possible to understand the strengths and weaknesses of workers, further develop their strengths and compensate for their weaknesses in the work, and at the same time fully motivate them.

In addition, this system can be used to assess the efficiency of workers, which is also useful for the recruitment and selection of workers. Candidates are given the necessary training and then asked to complete several tasks. Their performance is then assessed. This method is a test of the candidate's actual working capability. For the selection of incumbent workers, their performance assessment in the usual way can be used as a reference. Actual work is the best tool to test workers' working capability. The results of the assessment can provide a basis for the implementation of work-based distribution.

Workload distribution, bonus system, labor cost estimation, and assessment of work efficiency can all be used as a benchmark.

By using the system to continuously analyze the same worker, changes in daily work can be visualized. It is possible to follow the changes and growth status of a worker's proficiency. By analyzing the work of all workers, when the working capability of all workers generally rises, the work standard can be revised, and the standard working times can be adjusted appropriately to improve the overall efficiency of the production line balance.

Furthermore, this automatic work analysis system may provide a basis for product layout and worker assignment. Using the movement time calculated utilizing this system as a reference, the configuration of tools around the workers can be improved. The tool material is placed in a fixed position close to the worker so that the worker can get it in the shortest time. In terms of worker assignment, it echoes the content of Section 3.5. New workers are given standardized training based on work analysis, and then placed in the most appropriate process. Starting from the time of initial organization of processes, the working time of each process gradually changes over time from both proficiency and fatigue perspectives. The actual working time is expected to deviate from the planned value. Therefore, working time measurement is performed by this system to observe the difference between the planned and actual measured values of working time for each process. Based on the measurement results, the next working time is predicted. When the difference is confirmed to a certain extent, the worker assignment is adjusted in time to keep it in the best condition. Working time for each process is measured in the system based on acceleration and position, and the difference between the planned and measured values of working time for each process is frequently checked to see if it is in accordance with the original plan. This automatic work analysis system contributes to the re-evaluation of the work capability of each worker and the redesign of worker assignments.

For on-site managers, what is clear is which work is effective and which needs further improvement so that targeted optimization can be achieved. The purpose of work assessment is to test the level of understanding, execution, and effectiveness of the standardized work of workers, as well as whether effective follow-up and timely improvement are taken. Through the assessment and analysis, the manager can understand which parts are critical or not, after overviewing the whole work process. It can identify ineffective or wasteful personnel actions, streamline operations, reduce worker fatigue, and customize standard operations to make them easier and more efficient, and increase productivity. It can find out the idle time, remove unnecessary actions, and revise the action time standard. It also makes operations easier and more efficient and increases productivity.

## 4.5 Summary

In this chapter, an automatic work analysis method for assessing workers' performance was proposed. The system developed for the implementation of the proposed method included an ultrasonic network and a TC for measuring position and moving speed, together with an acceleration sensor for testing the acceleration speed of the dominant working hand of the worker during the working process. The ultrasonic network was used in assembly plants where the transmission and reception of ultrasonic signals are only slightly affected by obstacles. The TC was used to measure the position of the positioning target even in places where it is difficult to transmit and receive ultrasonic waves. The acceleration sensor measured acceleration in three axes.

Firstly, the method of position measurement using an ultrasonic sensor network was explained and formalized. Then, a new process for connecting a flow line measured by the ultrasonic sensor and a flow line measured by the TC was proposed. Position and moving speed were obtained by using the ultrasonic network and the TC. After that, based on position measurement data (i.e., position and moving speed), and the wavelet transform coefficients of acceleration, the most accurate estimation model was selected after using extensive cross-validation. Furthermore, an experiment was conducted to test the measurement accuracy of the proposed method. The motion estimation results were shown to be more than 90% consistent with the results obtained by video analysis. At last, the assessment of workers' performance based on work analysis was discussed.

# Chapter 5 Conclusion

## 5.1 Conclusive summary

This thesis addresses the issues of planning and management of worker assignment in a line production system from the perspective of worker optimal assignment and work analysis.

In terms of the worker optimal assignment, workers have different processing capabilities for different processes in the reset limited-cycle model with multiple periods was focused. First, a new formulation of the total expected cost analytically where each worker has a different capability for all processes was proposed as a theorem. Secondly, a method for calculating the total expected cost where there are two types of processing capabilities (proficient or inefficient) of a worker for each process was proposed. And then, based on the assumption that there are two types of workers' capabilities for each process, theorems of the rule of the optimal worker assignment were proposed in the following two cases. In the case where workers can be assigned to their proficient tasks, the optimal assignment is that every worker can be assigned to the process they are proficient at performing. This obvious thing was proved mathematically. In the case where an inefficient worker is assigned to only one task, but a proficient worker can be assigned to all remaining tasks, it is the optimal assignment that inefficient workers should be placed either at the starting point or at the ending point of the process that can assign the inefficient workers, and proficient workers should be positioned at the remaining points of the process. Furthermore, the validity of the theorem was confirmed through numerical experiments. Finally, a method for adjusting workers according to the difference between the planned and measured values by measuring the working time of each process was described as a method for staffing the worker based on optimal assignment rules.

This thesis considers the case of producing only one product with n processes. With inequality as the condition, optimal worker assignment can be obtained when the condition is satisfied. The optimal assignment rules obtained in this thesis apply to the design of production systems. The results can be used not only in the assignment of workers but also in the assignment of machines with different functions.

To find the difference between planned working time and actual working time, an automated work analysis method was suggested. The purpose of the work analysis system is to assist in going back and forth between worker assignment in the theoretical model and in practice. The system developed using the proposed method contained an ultrasonic network and a TC to measure the location and movement speed, as well as an acceleration sensor to test the worker's dominant working hand's acceleration speed. The ultrasonic network was utilized in assembly plants where barriers had little impact on the transmission and receipt of ultrasonic signals. The TC was used to measure the position of the positioning target even in places where it is difficult to transmit and receive ultrasonic waves. The acceleration sensor measured acceleration in three axes. The method of measuring position using an ultrasonic sensor network was first described and explained. Then, a new process for connecting a flow line measured by the ultrasonic sensor and a flow line measured by the TC was developed. The TC and ultrasonic network were used to determine the position and moving speed. Moreover, using extensive cross-validation, the most accurate estimation model was chosen based on position measurement data (i.e., position and moving speed), and the wavelet transforms coefficients of acceleration. Additionally, an experiment was conducted to test the measurement accuracy of the proposed method. The results of the motion estimation were found to be more than 90% consistent with those of the video analysis. The assessment of workers' performance based on work analysis was finally discussed.

The work analysis system measures the working time necessary for assessing work performance. By using time measurement and motion analysis with this system, work capability is assessed, thus helping managers quickly get an accurate picture of daily production. The actual processing capability of workers is used as a basis for optimizing the staffing of the assembly line. Furthermore, since the actual working time of each process changes according to each worker's degree of skill and the replacement of workers, etc., the work analysis automation method can measure the actual working time of each process and use the results in a re-evaluation of the work capability of each worker. Work capability is measured in practice on a time-by-time basis, and when the working time changes, the theoretical model is returned and workers are reassigned.

## 5.2 Future work

In terms of optimizing the worker assignment, there is no mathematical model that can take all the influencing factors into account because there are many factors that affect the balance of the assembly line. The more factors are considered, the more complex the theoretically designed model will be, the more operational elements and constraints in the mathematical model will become, and the more challenging it will be to find a suitable solution method. In future research, it still hopes to try to find out the rules, which are demonstrated by the case where there are not only two types of processing capabilities, but various ones. The types of workers to deal with more field situations will be increased. When designing the production lines, it is hoped that the method of setting the cycle time, the approach to setting the processing cost per unit time and cost per unit of time due to idle or delay will be discussed while considering different production lines. In addition, factors such as the learning efficiency of workers and differences between workers will be considered to introduce to enrich the current mathematical model and propose the optimal staffing solution from more perspectives, hoping to cope with more different situations.

In terms of the automated systems for work analysis, by increasing the number of acceleration sensors (e.g., adding it to the chest and waist), the recognition accuracy can be improved, and the number of recognizable movements can be increased. It intends to make them more practical for use by workers on-site.

The development of intelligent manufacturing has brought positive changes to the traditional manufacturing industry, industrial robots are gradually replacing traditional workers, and the production line is gradually moving from a purely manual line to a semi-automated line with a mixture of manual and machine, and even to a fully automated line in the future. In the future, it is hoped that the research of worker assignment can also advance from the all-manual production line to a semi-automated line with a mixture of manual and machine.

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